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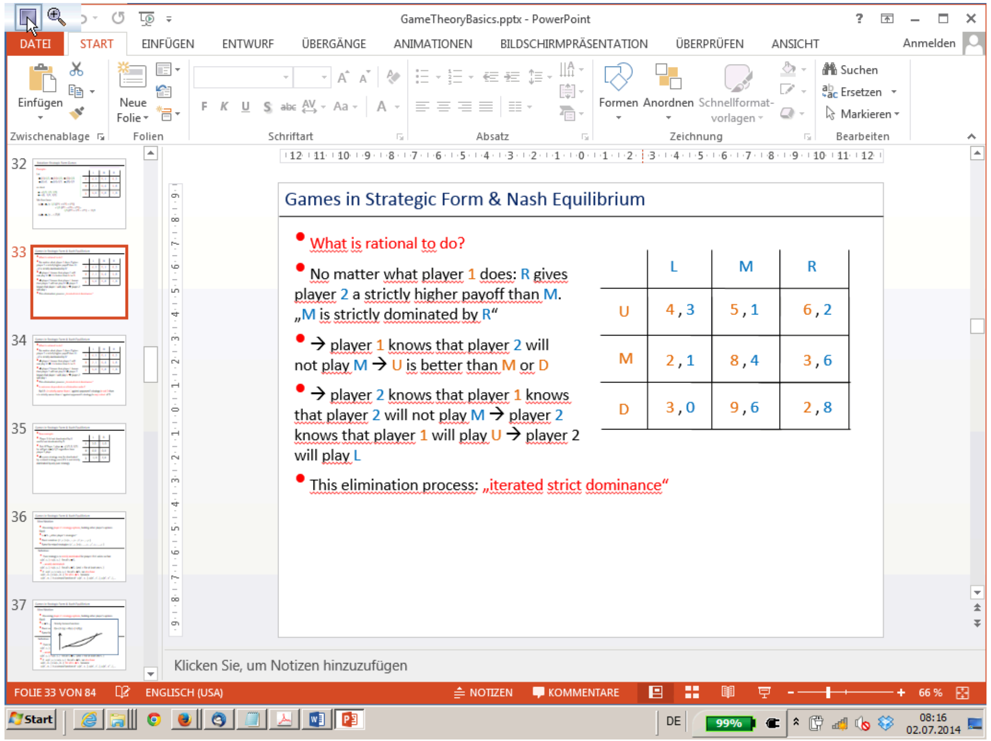

Games in Strategic Form & Nash Equilibrium

- New example:
- Player 1: M not dominated by U and M not dominated by D
- But: If Player 1 plays $\sigma_1 = (1/2, 0, 1/2)$ he will get $u(\sigma_1) = 1/2$ regardless how player 2 plays
- → a pure strategy may be dominated by a mixed strategy even if it is not strictly dominated by any pure strategy

	L	R
U	2, 0	-1, 0
M	0, 0	0, 0
D	-1, 0	2, 0

Games in Strategic Form & Nash Equilibrium

More Notation:

- Discussing player i's strategy-options, holding other player's options fixed:
- $s_{-i} \in S_{-i}$: „other player's strategies"
- Short notation: $(s'_i, s_{-i}) := (s_1, \dots, s_{i-1}, s'_i, s_{i+1}, \dots, s_I)$
- Same for mixed strategies: $(\sigma'_i, \sigma_{-i}) := (\sigma_1, \dots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \dots, \sigma_I)$

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- Pure strategy s_i is strictly dominated for player i if σ'_i exists so that $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$
- … weakly dominated: $u_i(\sigma'_i, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ (and > for at least one s_{-i})
- If $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ we also have $u_i(\sigma'_i, \sigma_{-i}) > u_i(s_i, \sigma_{-i})$ for all $\sigma_{-i} \in S_{-i}$ because $u_i(\sigma'_i, \sigma_{-i})$ is a convex function of $u_i(\sigma'_i, s_{-i}), u_i(\sigma'_i, s'_{-i}), u_i(\sigma'_i, s''_{-i}), \dots$

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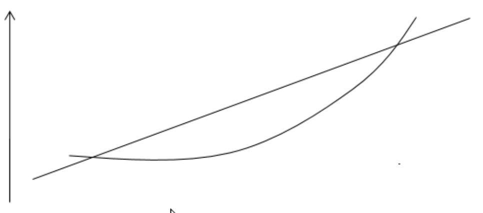
- Same fo

Definition:

- Pure st
$u_i(\sigma'_i, s_{-i})$ >
- … weak

Strictly Convex function:

$f(tx+(1-t)y) < tf(x) + (1-t)f(y)$



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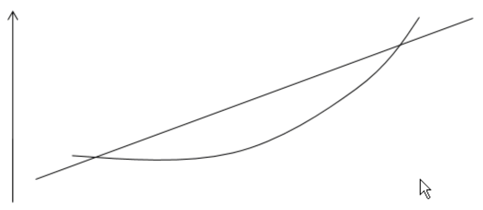

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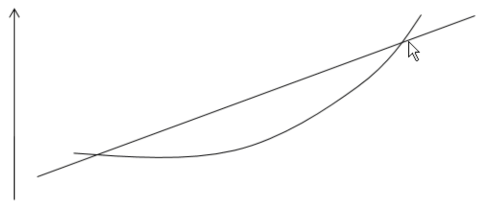

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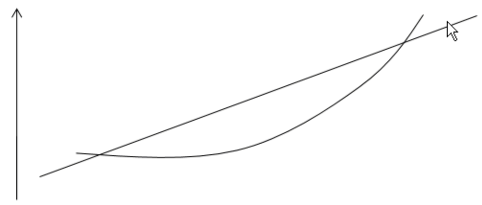

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Games in Strategic Form & Nash Equilibrium

- What about dominated mixed strategies?
- Easy: A mixed strategy that assigns positive probabilities to pure strategies that are dominated is dominated
- But: A mixed strategy may be dominated even if it assigns positive probabilities to pure strategies that are not even weakly dominated:

	L	R
U	1, 3	-2, 0
M	-2, 0	1, 3
D	0, 1	0, 1

Example:

- U and M are not dominated by D for player 1
- But: Playing $\sigma_1=(½, ½, 0)$ gives expected utility $u_1(\sigma_1, *) = -1/2$ no matter what 2 plays → D ($\sigma_D=(0, 0, 1)$) dominates σ_1

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A note on rationality

	L	R
U	8, 10	-100, 9
D	7, 6	6, 5

- Iterated strict dominance → (U,L)
- BUT: psychology → play D instead of U because „U is unsafe"

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Game Theory ←→ Decision Theory

- Example

- Iterated strict dominance → (U,L)

	L	R
U	1, 3	4, 1
D	0, 2	3, 4

- If player 1 reduces his payoff for U by 2:
  - decison theory: no use
  - game theory: new iterated strict dominance → (D,R)

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U	-1, 3	2, 1
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Games in Strategic Form & Nash Equilibrium

Prisoner's dilemma & Iterated dominance

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

- Iterated strict dominance → (D,D)

Games in Strategic Form & Nash Equilibrium

Prisoner's dilemma & Iterated dominance

	C	D
C	1, 1	-1, 2
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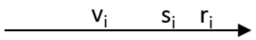
- Iterated strict dominance → (D,D)

Games in Strategic Form & Nash Equilibrium

Vickrey Auction & Iterated dominance

- Good‘s valuations: v_i ; Assume common knowledge for the moment
- Bids: s_i
- Second price:
  - winning condition: $s_i > \max_{j\neq i} s_j$
  - let $r_i := \max_{j\neq i} s_j$ r_i is the price having to be paid
  - winner i´s utility: $u_i = v_i - r_i$; other players utility = 0

- for each player bidding true valuation is weakly dominant:
  - case $s_i > v_i$: (overbidding)
    - If $r_i > s_i$: looses → $u_i = 0$
      → could have bidden v_i as well



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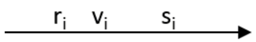

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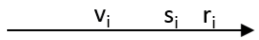

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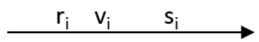

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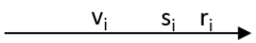

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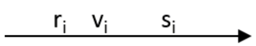

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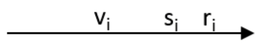

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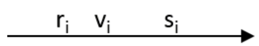

Vickrey Auction & Iterated dominance

- Good's valuations: v_i ; Assume common knowledge for the moment
- Bids: s_i
- Second price:
  - winning condition: $s_i > \max_{j \neq i} s_j$
  - let $r_i := \max_{j \neq i} s_j$ r_i is the price having to be paid
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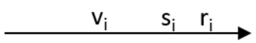

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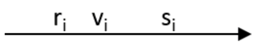

Games in Strategic Form & Nash Equilibrium

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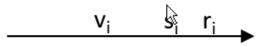

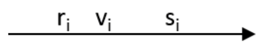

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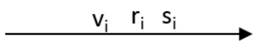

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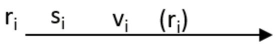

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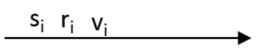

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Games in Strategic Form & Nash Equilibrium

Vickrey Auction & Iterated dominance

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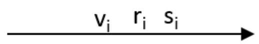

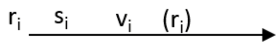

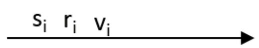

Vickrey Auction & Iterated dominance

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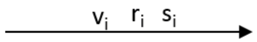

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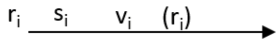

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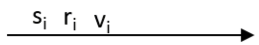

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Games in Strategic Form & Nash Equilibrium

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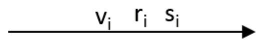

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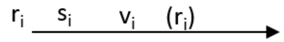

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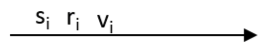

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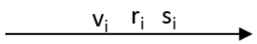

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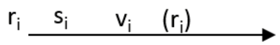

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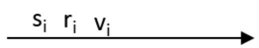

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium

- Nash Equilibrium : strategy profile: each player's strategy is optimal response to all other player's strategies:

- Mixed strategy profile σ^* is Nash Equilibrium if

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  (Pure strategy profiles also possible → „pure strategy NE")

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→ in NE def we must only check for pure alternatives s_i

→ In a (non-degenerate) mixed strategy Nash Equilibrium a player must be (a priori) indifferent between all pure strategies to which he assigns positive probability (Indifference condition)

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Indifference condition: more detailed explanation:

For player I's utility, we have:

$$u_i(\sigma) = \sum_{s_i \in S_u} \sigma_i(s_i) u_i(s_i, \sigma_{-i}) \quad \text{with} \quad \sum_{s_i \in S_u} \sigma_i(s_i) = 1$$

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- In reality: If rationality is „non-strict“ (mistakes are made): deviations can occur
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Nash Equilibrium: Example: Cournot Competition

- Cournot model: Duopoly. Each of two firms (players) i produces same good.
- Output levels q_i are chosen from sets Q_i
- Cost of production is $c_i(q_i)$
- Market price is $p(q) = p(q_1+q_2)$
- Firm i's profit is then $u_i(q_1, q_2) = q_i\, p(q) - c_i(q_i)$
- Cournot reaction functions $r_1 : Q_2 \rightarrow Q_1$ and $r_2 : Q_1 \rightarrow Q_2$ specify optimal reaction on output level of opponent

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- Under certain reasonable assumptions (see [1]) we can maximize e.g. $u_2(q_1, q_2)$ by solving $d/dq_2\; u_2(q_1, q_2) = 0$ which yields

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- Two firms, 1 (at x=0) and 2 (at x=1) sell same good
- Unit cost of product := c; price for product of firm i := p_i
- Customers: uniformly distributed over [0,1] with probability density 1
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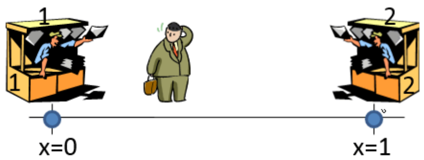

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- Demand for firm 1 is $D_1(p_1,p_2) = x$ where $p_1+tx = p_2+t(1-x)$
- → $D_1(p_1,p_2) = (p_2-p_1+t) / (2t)$
- $D_1(p_1,p_2) = 1 – D_2(p_1,p_2)$
- Nash Equilibirium (p^*_1,p^*_2): For each i: $p^*_i \in \text{argmax} \{(p_i - c) D_i(p_i, p^*_{-i})\}$
- Denoting the reaction functions by $r_1(p_2)$ and $r_2(p_1)$ we get for e.g. firm 2:

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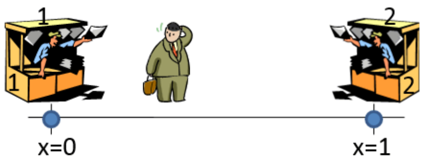

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence-of Pure NE-Example

- Some games may have more than one pure strategy NE
- Not all games have a pure strategy NE:
- Example: Matching pennies:
- Both players simultaneously announce
Head or Tails: IF match → 1 wins; If differ → 2 wins
- No pure NE;
but mixed strategy NE: ((1/2, 1/2); (1/2, 1/2)) :

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

- Reasoning: If player 2 plays (1/2, 1/2) then player 1's expected payoff is ½ *1 + ½ *(-1) = 0 when playing H and ½ *(-1) + ½ *1 = 0 when playing T → player 1 is also indifferent

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence--of Pure NE-Example 2

- Another example: Inspection game
- Worker: work or shirk; Employer: Inspect or not inspect
- Worker: working costs g, produces value v; gets wage w
- Employer: Inspection costs h
- We assume w > g > h > 0
- If not inspect → worker shirks → better inspect → if inspect → worker always works → better not inspect → ...: No pure NE
- → Employer must randomize

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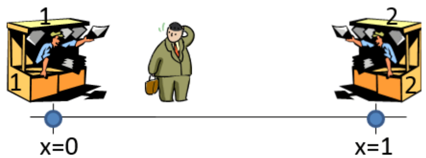

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    → $g = yw$ → $y=g/w$

  - → For employer indifferent between I and NI: inspection costs == expctd. wage savings:

    $x(-h)+(1-x)(v-w-h) = x(-w) + (1-x)(v-w)$

    → $h = xw$ → $x= h/w$

	I	NI
S	0,-h	w,-w
W	w-g, v-w-h	w-g, v-w

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

- Another example: Battle of the sexes
- Man & Woman; Ballet or Football

	B	F
F	0, 0	2, 1
B	1, 2	0, 0

- Another example: Game of chicken
- Driver 1 & Driver 2; Tough or Weak

	T	W
T	-1,-1	2, 1
W	1, 2	0, 0

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

- Another example: Battle of the sexes
- Two pure NE: (F;F) and (B;B)
- One mixed NE: Indifference condition
→ Let $\sigma_1(F)=x$ and $\sigma_2(B)=y$ →
Player 1's indifference:
$0\,y + 2(1-y) = 1\,y + 0\,(1-y)$ → $y=2/3$
Player 2's indifference:
$0\,x + 2(1-x) = 1\,x + 0\,(1-x)$ → $x=2/3$
→ Mixed NE: ((2/3, 1/3); (2/3, 1/3))

	B	F
F	0, 0	2, 1
B	1, 2	0, 0

- Another example: Game of chicken
- (same reasoning) →
Mixed NE: ((1/2, 1/2); (1/2, 1/2))

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Focal points

- Some games have more than one NE → which will be chosen?
- Theory of „focalness" of NE („focal points"):
Example: Chose time of day simultaneously; reward if match: 12 noon is focal, 15:37 is not

Risk Dominance

- Stag Hunt: NE: (C;C) and (D;D); (C;C) is pareto-dominant → (C;C) might be chosen if p(C)>0.5
BUT
- more than two players: ALL have to agree on C
→ $p(C)^8 > 0.5$ → $p(C) > 0.93$ → (D;D) „risk dominates" (C;C)

	Hunt Stag (C)	Hunt Hare (D)
Hunt Stag (C)	2 , 2	0 , 1
Hunt Hare (D)	1 , 0	1 , 1

Games in Strategic Form & Nash Equilibrium

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Risk Dominance / Pareto Optimality

	L	R
U	9, 9	0, 8
D	8, 0	7, 7

- In this game: (Among others) two pure NE: (U,L) and (D,R); (U,L): Pareto dominates (D,R)
- But: For player 1 D is safer (guarrantees min payoff of 7) → If p(R) > 1/8 don't go for (U,L) → no certainty!
- Pregame-communication / agreement on (U,L) ?!
No: player 2 gains if player 1 plays U → player 2 will always tell „L“ regardless of true intentions → agreement is worthless

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Risk Dominance / Pareto Optimality

	L	R
U	0,0,10	-5,-5,0
D	-5,-5,0	1,1,-5

A

	L	R
U	-2,-2,0	-5,-5,0
D	-5,-5,0	-1,-1,5

B

- Three player game: Two pure NE: (U,L,A) and (D,R,B); (and one mixed) ; (U,L,A) pareto-dominates (D,R,B)
- If player 3´s choice is fixed→ Two player game → (D,R) is pareto-dominant → if players 1 and 2 expect A : coordinate on (D,R).
- → concept of „coalition proof eq.“ (here (D,R,B))(see [1])

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE
Risk Dominance / Pareto Optimality

	L	R
U	0,0,10	-5,-5,0
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Games in Strategic Form & Nash Equilibrium

Mixed Nash Equilibrium: General Analysis for 2 x 2 Games (see [2])

		Player B: q	1-q
		L	R
Player A: p	U	a_{UL}, b_{UL}	a_{UR}, b_{UR}
1-p	D	a_{DL}, b_{DL}	a_{DR}, b_{DR}

- Pure NE: One cell →
For A: cell's payoff for A must be (weak) maximum over rows in that column
For B: cell's payoff for B must be (weak) maximum over column in that row
- Example: (U,R) is pure NE if $a_{UR} \geq a_{DR}$ and $b_{UR} \geq b_{UL}$

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Mixed Nash Equilibrium: General Analysis for 2 x 2 Games (see [2])

		Player B: q	1-q
		L	R
Player A: p	U	a_{UL}, b_{UL}	a_{UR}, b_{UR}
1-p	D	a_{DL}, b_{DL}	a_{DR}, b_{DR}

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For A: cell's payoff for A must be (weak) maximum over rows in that column
For B: cell's payoff for B must be (weak) maximum over column in that row
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