Script generated by TTT

Title: Seidl: Virtual Machines (19.05.2014)

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Pages: 26

Example: **let** $a = 17$ **in let** $f =$ **fun** $b \rightarrow a + b$ **in** f 42

Disentanglement of the jumps produces:

0	loadc 17	2	mark B	3	B:	slide 2	1	pushloc 1
1	mkbasic	5	loadc 42	1		halt	2	eval
1	pushloc 0	6	mkbasic	0	A:	targ 1	2	getbasic
2	mkvec 1	6	pushloc 4	0		pushglob 0	2	add
2	mkfunval A	7	eval	1		eval	1	mkbasic
		7	apply	1		getbasic	1	return 1

(1, fun x → x+1)

24 Structured Data

In the following, we extend our functional programming language by some datatypes.

24.1 Tuples

Constructors: $(.,\ldots,.)$, k-ary with $k \geq 0$;

Destructors: $\#j$ for $j \in \mathbb{N}_0$ (Projections)

We extend the syntax of expressions correspondingly:

$$e \; ::= \; \ldots \mid (e_0, \ldots, e_{k-1}) \mid \#j\, e$$
$$\mid \; \textbf{let}\ (x_0, \ldots, x_{k-1}) = e_1\ \textbf{in}\ e_0$$

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- In order to construct a tuple, we collect sequence of references on the stack. Then we construct a vector of these references in the heap using mkvec
- For returning components we use an indexed access into the tuple.

$$\begin{array}{lcl} \text{code}_V\ (e_0,\ldots,e_{k-1})\ \rho\ \text{sd} & = & \text{code}_C\ e_0\ \rho\ \text{sd} \\ & & \text{code}_C\ e_1\ \rho\ (\text{sd}+1) \\ & & \ldots \\ & & \text{code}_C\ e_{k-1}\ \rho\ (\text{sd}+k-1) \\ & & \text{mkvec k} \end{array}$$

$$\begin{array}{lcl} \text{code}_V\ (\#j\ e)\ \rho\ \text{sd} & = & \text{code}_V\ e\ \rho\ \text{sd} \\ & & \text{get j} \\ & & \text{eval} \end{array}$$

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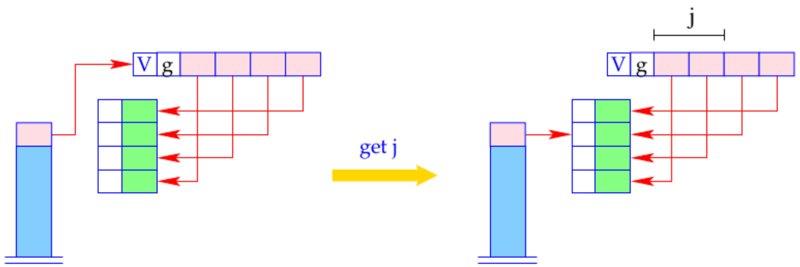


```
if (S[SP] == (V,g,v))
   S[SP] = v[j];
else Error "Vector expected!";
```


Inversion: Accessing all components of a tuple simulataneously:

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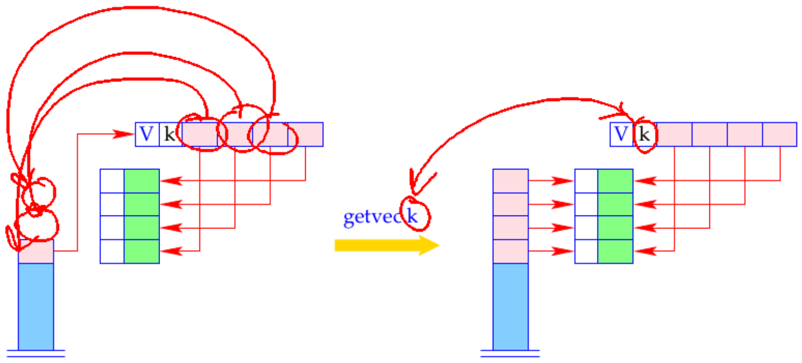
This is translated as follows:

$$\begin{array}{lcl} \text{code}_V\ e\ \rho\ \text{sd} & = & \text{code}_V\ e_1\ \rho\ \text{sd} \\ & & \text{getvec k} \\ & & \text{code}_V\ e_0\ \rho'\ (\text{sd}+k) \\ & & \text{slide k} \end{array}$$

where $\rho' = \rho \oplus \{y_i \mapsto (L, sd+i+1) \mid i = 0,\ldots,k-1\}$.

The instruction getvec k pushes the components of a vector of length k onto the stack:




```
if (S[SP] == (V,k,v)) {
  SP--;
  for(i=0; i<k; i++) {
      SP++; S[SP] = v[i];
  }
} else Error "Vector expected!";
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24.2 Lists

Lists are constructed by the constructors:

[] "Nil", the empty list;

"::" "Cons", right-associative, takes an element and a list.

Access to list components is possible by **match**-expressions ...

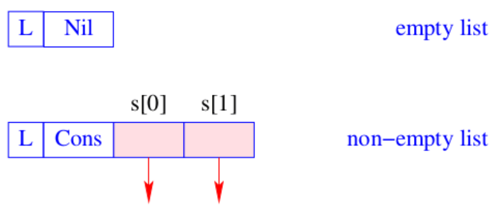
Example: The append function app:

$$\begin{aligned} \text{app} \quad = \quad & \textbf{fun } l\; y \to \textbf{match } l \textbf{ with} \\ & \mid\; [\,] \to y \\ & \mid\; h :: t \to h :: (\text{app } t\; y) \end{aligned}$$

accordingly, we extend the syntax of expressions:

$$\begin{aligned} e \quad ::= \quad & \ldots \mid [\,] \mid (e_1 :: e_2) \\ & \mid (\textbf{match } e_0 \textbf{ with } [\,] \to e_1 \mid h :: t \to e_2) \end{aligned}$$

Additionally, we need new heap objects:



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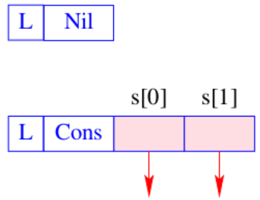

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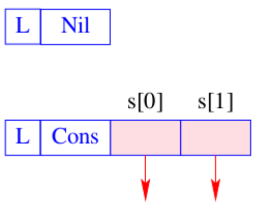

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24.3 Building Lists

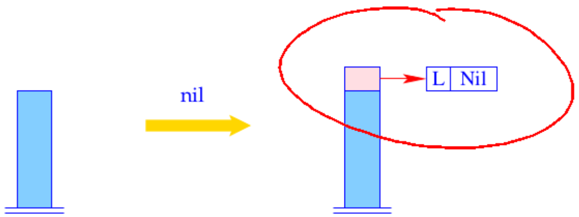
The new instructions nil and cons are introduced for building list nodes.

We translate for CBN:

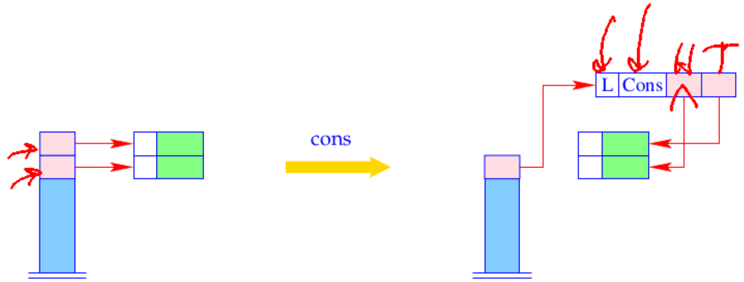
$$\begin{array}{lll} \text{code}_V\ [\,]\ \rho\ \text{sd} & = & \text{nil} \\ \text{code}_V\ (e_1 :: e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\ & & \text{code}_C\ e_2\ \rho\ (\text{sd}+1) \\ & & \text{cons} \end{array}$$

Note:

- With CBN: Closures are constructed for the arguments of “::”;
- With CBV: Arguments of “::” are evaluated :-)



SP++; S[SP] = new (L,Nil);



S[SP-1] = new (L,Cons, S[SP-1], S[SP]);
SP- -;

24.4 Pattern Matching

Consider the expression $e \equiv \textbf{match}\ e_0\ \textbf{with}\ [\,] \to e_1 \mid h :: t \to e_2$.

Evaluation of e requires:

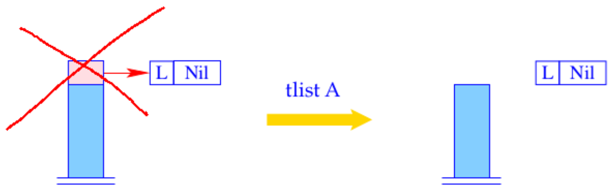
- evaluation of e_0;
- check, whether resulting value v is an L-object;
- if v is the empty list, evaluation of e_1 ...
- otherwise storing the two references of v on the stack and evaluation of e_2. This corresponds to binding h and t to the two components of v.

In consequence, we obtain (for CBN as for CBV):

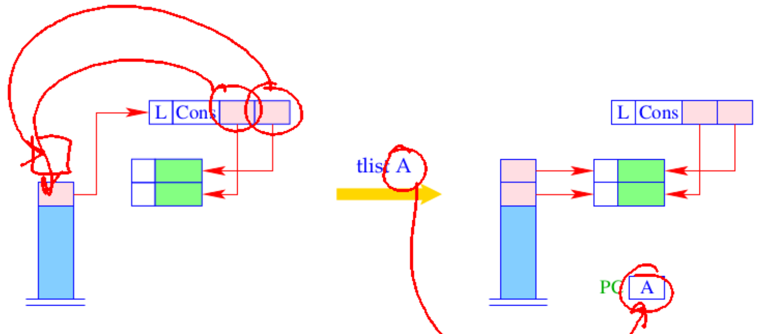
$$
\begin{array}{rcll}
\text{code}_V\ e\ \rho\ \text{sd} & = & & \text{code}_V\ e_0\ \rho\ \text{sd} \\
& & & \text{tlist A} \\
& & & \text{code}_V\ e_1\ \rho\ \text{sd} \\
& & & \text{jump B} \\
& & \text{A}: & \text{code}_V\ e_2\ \rho'\ (\text{sd}+2) \\
& & & \text{slide } 2 \\
& & \text{B}: & \ldots
\end{array}
$$

where $\rho' = \rho \oplus \{h \mapsto (L, sd+1), t \mapsto (L, sd+2)\}$.

The new instruction tlist A does the necessary checks and (in the case of Cons) allocates two new local variables:



h = S[SP];
if (H[h] != (L,...))
    Error "no list!";
if (H[h] == (_,Nil)) SP- -;
    ...



... else {
    S[SP+1] = S[SP]→s[1];
    S[SP] = S[SP]→s[0];
    SP++; PC = A;
}

Example: The (disentangled) body of the function app with app $\mapsto (G, 0)$:

0		targ 2	3		pushglob 0	0	C:	mark D
0		pushloc 0	4		pushloc 2	3		pushglob 2
1		eval	5		pushloc 6	4		pushglob 1
1		tlist A	6		mkvec 3	5		pushglob 0
0		pushloc 1	4		mkclos C	6		eval
1		eval	4		cons	6		apply
1		jump B	3		slide 2	1	D:	update
2	A:	pushloc 1	1	B:	return 2			

Note:

Datatypes with more than two constructors need a generalization of the tlist instruction, corresponding to a `switch`-instruction :-)

24.5 Closures of Tuples and Lists

The general schema for code_C can be optimized for tuples and lists:

$$\begin{aligned}
\text{code}_C\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} &= \text{code}_V\ (e_0, \ldots, e_{k-1})\ \rho\ \text{sd} = \text{code}_C\ e_0\ \rho\ \text{sd} \\
&\qquad \text{code}_C\ e_1\ \rho\ (\text{sd}+1) \\
&\qquad \ldots \\
&\qquad \text{code}_C\ e_{k-1}\ \rho\ (\text{sd}+k-1) \\
&\qquad \text{mkvec k} \\
\text{code}_C\ []\ \rho\ \text{sd} &= \text{code}_V\ []\ \rho\ \text{sd} = \text{nil} \\
\text{code}_C\ (e_1 :: e_2)\ \rho\ \text{sd} &= \text{code}_V\ (e_1 :: e_2)\ \rho\ \text{sd} = \text{code}_C\ e_1\ \rho\ \text{sd} \\
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 & & & & \text{code}_C\ e_1\ \rho\ (\text{sd}+1) \\
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 & & & & \text{mkvec k} \\
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\text{code}_C\ (e_1 :: e_2)\ \rho\ \text{sd} & = & \text{code}_V\ (e_1 :: e_2)\ \rho\ \text{sd} & = & \text{code}_C\ e_1\ \rho\ \text{sd} \\
 & & & & \text{code}_C\ e_2\ \rho\ (\text{sd}+1) \\
 & & & & \text{cons}
\end{array}
$$

25 Last Calls

A function application is called last call in an expression e if this application could deliver the value for e.

A function definition is called tail recursive if all recursive calls are last calls.

Examples:

$r\ t\ (h :: y)$ is a last call in **match** x **with** $[] \to y \mid h :: t \to r\ t\ (h :: y)$

$f\ (x-1)$ is not a last call in **if** $x \leq 1$ **then** 1 **else** $x * f\ (x-1)$

Observation: Last calls in a function body need no new stack frame!

$\Longrightarrow$

Automatic transformation of tail recursion into loops!!!

The code for a last call $l \equiv (e'\ e_0 \ldots e_{m-1})$ inside a function f with k arguments must

1. allocate the arguments e_i and evaluate e' to a function (note: all this inside f's frame!);
2. deallocate the local variables and the k consumed arguments of f;
3. execute an apply.

$$
\begin{array}{lll}
\text{code}_V\ l\ \rho\ \text{sd} & = & \text{code}_C\ e_{m-1}\ \rho\ \text{sd} \\
 & & \text{code}_C\ e_{m-2}\ \rho\ (\text{sd}+1) \\
 & & \ldots \\
 & & \text{code}_C\ e_0\ \rho\ (\text{sd}+m-1) \\
 & & \text{code}_V\ e'\ \rho\ (\text{sd}+m) \qquad \text{// Evaluation of the function} \\
 & & \text{move}\ r\ (m+1) \qquad \text{// Deallocation of } r \text{ cells} \\
 & & \text{apply}
\end{array}
$$

where $r = \text{sd} + k$ is the number of stack cells to deallocate.