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Games in Strategic Form & Nash Equilibrium

More Notation:

- Discussing player i's strategy-options, holding other player's options fixed:
- $s_{-i} \in S_{-i}$: „other player's strategies"
- Short notation: $(s'_i, s_{-i}) := (s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_I)$
- Same for mixed strategies: $(\sigma'_i, \sigma_{-i}) := (\sigma_1, \ldots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \ldots, \sigma_I)$

Definition:

- Pure strategy s_i is strictly dominated for player i if σ'_i exists so that
$u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$
- ... weakly dominated:
$u_i(\sigma'_i, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ (and $>$ for at least one s_{-i})
- If $u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in S_{-i}$ we also have
$u_i(\sigma'_i, \sigma_{-i}) > u_i(s_i, \sigma_{-i})$ for all $\sigma_{-i} \in S_{-i}$ because
$u_i(\sigma'_i, \sigma_{-i})$ is a convex function of $u_i(\sigma'_i, s_{-i}), u_i(\sigma'_i, s'_{-i}), u_i(\sigma'_i, s''_{-i}), \ldots$

Notation: Strategic Form Games

- Two Player zero sum game: $$\forall s : \sum_{i=1}^{2} u_i(s) = 0$$
- Structure of game is common knowledge:
all players know;
all players know that all players know;
all players know that all players know that all players know;
....
- Mixed strategy $\sigma_i : S_i \rightarrow [0,1]$ Probability distribution over pure strategies (statistically independent for each player);
Examples: $\sigma_1(U)=1/3$, $\sigma_1(M)=2/3$, $\sigma_1(D)=0$;
$\sigma'_1(U)=2/3$, $\sigma'_1(M)=1/6$, $\sigma'_1(D)=1/6$;
.....
- Thus: $\sigma_i(s_i)$ is the probability that player i assigns to strategy (action) s_i

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
D	3 , 0	9 , 6	2 , 8

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Notation: Strategic Form Games

Example:

Let

$\sigma_1(U)=1/3$, $\sigma_1(M)=1/3$, $\sigma_1(D)=1/3$
$\sigma_2(L)=0$, $\sigma_2(M)=1/2$, $\sigma_2(R)=1/2$

or short

$\sigma_1 = (1/3, 1/3, 1/3)$
$\sigma_2 = (0, 1/2, 1/2)$

	L	M	R
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We then have:

$u_1(\sigma_1, \sigma_2) = 1/3\,(0*4 + ½*5 + ½*6) + 1/3\,(0*2 + ½*8 + ½*3) + 1/3\,(0*3 + ½*9 + ½*2) = 11/2$

$u_2(\sigma_1, \sigma_2) = \ldots = 27/6$

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$u_2(\sigma_1, \sigma_2) = \ldots = 27/6$

Games in Strategic Form & Nash Equilibrium

- What is rational to do?
- No matter what player 1 does: R gives player 2 a strictly higher payoff than M. „M is strictly dominated by R“
- → player 1 knows that player 2 will not play M → U is better than M or D
- → player 2 knows that player 1 knows that player 2 will not play M → player 2 knows that player 1 will play U → player 2 will play L

	L	M	R
U	4 , 3	5 , 1	6 , 2
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- This elimination process: „iterated strict dominance“
- Is outcome dependent on elimination order?

No! If s_i is strictly worse than s_i' against opponent‘s strategy in set D then s_i is strictly worse than s_i' against opponent‘s strategy in any subset of D

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- What about dominated mixed strategies?
- Easy: A mixed strategy that assigns positive probabilities to pure strategies that are dominated is dominated
- But: A mixed strategy may be dominated even if it assigns positive probabilities to pure strategies that are not even weakly dominated:

Example:

- U and M are not dominated by D for player 1
- But: Playing $\sigma_1 = (½, ½, 0)$ gives expected utility $u_1(\sigma_1, *) = -1/2$ no matter what 2 plays → D ($\sigma_D = (0, 0, 1)$) dominates σ_1

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A note on rationality

	L	R
U	8, 10	-100, 9
D	7, 6	6, 5

- Iterated strict dominance → (U,L)
- BUT: psychology → play D instead of U because „U is unsafe“

Games in Strategic Form & Nash Equilibrium

A note on rationality

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Game Theory ←→ Decision Theory

- Example

- Iterated strict dominance → (U,L)

	L	R
U	1, 3	4, 1
D	0, 2	3, 4

↓

	L	R
U	-1, 3	2, 1
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- If player 1 reduces his payoff for U by 2:
  - decison theory: no use
  - game theory: new iterated strict dominance → (D,R)

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Game Theory ←→ Decision Theory

- Example

- Iterated strict dominance → (U,L)

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Games in Strategic Form & Nash Equilibrium

Prisoner's dilemma & Iterated dominance

	C	D
C	1, 1	-1, 2
D	2, -1	0, 0

- Iterated strict dominance → (D,D)

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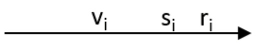
- Good's valuations: v_i ; Assume common knowledge for the moment
- Bids: s_i
- Second price:
  - winning condition: $s_i > \max_{j \neq i} s_j$
  - let $r_i := \max_{j \neq i} s_j$ r_i is the price having to be paid
  - winner i´s utility: $u_i = v_i - r_i$; other players utility = 0
- for each player bidding true valuation is weakly dominant:
  - case $s_i > v_i$: (overbidding)
    - If $r_i > s_i$: looses → $u_i = 0$
      → could have bidden v_i as well



    - If $r_i \leq v_i$: wins → $u_i = v_i - r_i$
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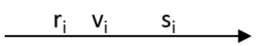

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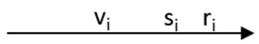
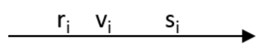

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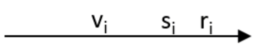
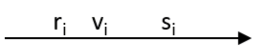

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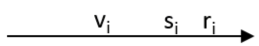
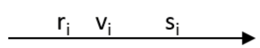

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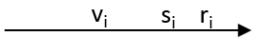

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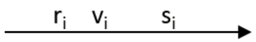

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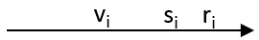

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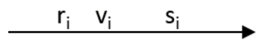

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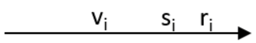

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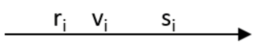

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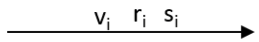
Vickrey Auction & Iterated dominance



- case $v_i < r_i < s_i$:
  - i wins → $u_i = v_i - r_i < 0$ (winner's curse)
    → should have bidden $v_i = r_i$ → $u_i = 0$ at least
- case $s_i < v_i$: (underbidding)
  - If $r_i \leq s_i$ or $r_i \geq v_i$:
    u_i is unchanged if he
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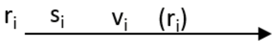

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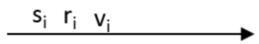

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Games in Strategic Form & Nash Equilibrium

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Nash Equilibrium

- Nash Equilibrium : strategy profile: each player‘s strategy is optimal response to all other player‘s strategies:

- Mixed strategy profile σ^* is Nash Equilibrium if

for all i: $u_i(\sigma^*_i, \sigma^*_{-i}) \geq u_i(s_i, \sigma^*_{-i})$ for all $s_i \in S_i$

(Pure strategy profiles also possible → „pure strategy NE“)

- Strategy profile s^* is Strict Nash Equilibrium: if it is a NE and

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→ In a (non-degenerate) mixed strategy Nash Equilibrium a player must be (a priori) indifferent between all pure strategies to which he assigns positive probability (Indifference condition)

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Indifference condition: more detailed explanation:

For player I's utility, we have:

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Notation: Strategic Form Games

- Space of mixed strategies for player i: Σ_i
- Space of mixed strategy profiles: $\Sigma = \times_i \Sigma_i$
- Mixed strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_I) \in \Sigma$
- Player i's payoff when a mixed strategy profile σ is played is

$$\sum_{s \in S} \left(\prod_{j=1}^{I} \sigma_j(s_j) \right) u_i(s)$$

denoted as $u_i(\sigma)$, is a linear function of the σ_i

- A pure strategy of a player is a special mixed strategy of that player with one probability equal to 1 and all others equal to 0

Games in Strategic Form & Nash Equilibrium

- What is rational to do?
- No matter what player 1 does: R gives player 2 a strictly higher payoff than M. „M is strictly dominated by R"
- → player 1 knows that player 2 will not play M → U is better than M or D
- → player 2 knows that player 1 knows that player 2 will not play M → player 2 knows that player 1 will play U → player 2 will play L

	L	M	R
U	4 , 3	5 , 1	6 , 2
M	2 , 1	8 , 4	3 , 6
D	3 , 0	9 , 6	2 , 8

- This elimination process: „iterated strict dominance"
- Is outcome dependent on elimination order?

  No! If s_i is strictly worse than s_i' against opponent's strategy in set D then s_i is strictly worse than s_i' against opponent's strategy in any subset of D

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- Strict equilibria need not exist. However each finite strategy form game has a mixed strategy equilibrium.
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- In reality: If rationality is „non-strict“ (mistakes are made): deviations can occur
- If one round of elimination of strictly dominated strategies yields unique strategy profile, this strategy profile is a strict NE (unique)
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Nash Equilibrium: Example: Cournot Competition

- Cournot model: Duopoly. Each of two firms (players) i produces same good.
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- Cost of production is $c_i(q_i)$
- Market price is $p(q) = p(q_1+q_2)$
- Firm i's profit is then $u_i(q_1, q_2) = q_i\, p(q) - c_i(q_i)$
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$d/dq_2\; [q_2\, p(q_1, q_2) - c_2(q_2)] = p(q_1, q_2) + p'(q_1, q_2)\, q_2 - c_2'(q_2) = 0.$

Inserting $r_2(q_1)$ for q_2

$p(q_1 + r_2(q_1)) + p'(q_1 + r_2(q_1))\, r_2(q_1) - c_2'(r_2(q_1)) = 0$

gives the defining equation for $r_2(.)$.

(analogous for $r_1(.)$).

- The intersections of the functions r_2 and r_1 are the NE of the Cournot game.
- Example: Linear demand $p(q) = \max(0, 1-q)$; linear cost: $c_i(q_i) = c\, q_i$:

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Games in Strategic Form & Nash Equilibrium

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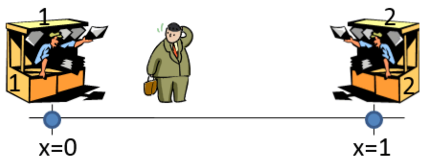

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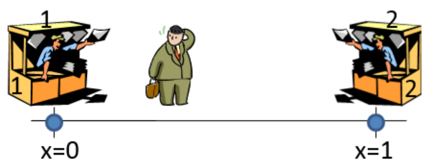

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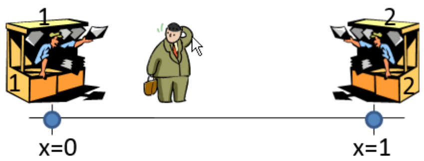

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence-of Pure NE-Example

- Some games may have more than one pure strategy NE
- Not all games have a pure strategy NE:
- Example: Matching pennies:
- Both players simultaneously announce
  Head or Tails: IF match → 1 wins; If differ → 2 wins
- No pure NE;
  but mixed strategy NE: ((1/2, 1/2); (1/2, 1/2)) :

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

- Reasoning: If player 2 plays (1/2, 1/2) then player 1's expected payoff is ½ *1 + ½ *(-1) = 0 when playing H and ½ *(-1) + ½ *1 = 0 when playing T → player 1 is also indifferent

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- Another example: Inspection game
- Worker: work or shirk; Employer: Inspect or not inspect
- Worker: working costs g, produces value v; gets wage w
- Employer: Inspection costs h
- We assume w > g > h > 0
- If not inspect → worker shirks → better inspect → if inspect → worker always works → better not inspect → ...: No pure NE
- → Employer must randomize

	I	NI
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Indifference condition: more detailed explanation:

For player I's utility, we have:

$$u_i(\sigma) = \sum_{s_i \in S_u} \sigma_i(s_i) u_i(s_i, \sigma_{-i}) \quad \text{with} \quad \sum_{s_i \in S_u} \sigma_i(s_i) = 1$$

for the NE σ^* we thus have:

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since $u_i(\sigma^*)$ is the best outcome , i can achieve, when the others play σ^*_{-i} , all the $u_i(s_i, \sigma^*_{-i})$ with $\sigma_i(s_i) > 0$ must be equal,
and equal to $u_i(\sigma^*)$.

why? → no $u_i(s_i, \sigma^*_{-i})$ can be greater than $u_i(\sigma^*)$ otherwise the NE condition would be violated, and also not smaller, because then the sum would also be smaller.

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$$u_i(\sigma^*) = \sum_{s_i \in S_u} \sigma_i^*(s_i) u_i(s_i, \sigma^*_{-i}) \quad \text{with} \quad \sum_{s_i \in S_u} \sigma_i^*(s_i) = 1$$

since $u_i(\sigma^*)$ is the best outcome , i can achieve, when the others play σ^*_{-i} , all the $u_i(s_i, \sigma^*_{-i})$ with $\sigma_i(s_i) > 0$ must be equal,
and equal to $u_i(\sigma^*)$.

why? → no $u_i(s_i, \sigma^*_{-i})$ can be greater than $u_i(\sigma^*)$ otherwise the NE condition would be violated, and also not smaller, because then the sum would also be smaller.

Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: Non-Existence--of Pure NE-Example 2

- If worker plays (x, 1-x) and employer plays (y, 1-y)
- Indifference condition in mixed strategy NE →
  - → For worker indifferent between S and W : gain from shirking == expected income loss:

    $0y+(1-y)w=y(w-g)+(1-y)(w-g)$

    → $g = yw$ → $y=g/w$
  - → For employer indifferent between I and NI: inspection costs == expctd. wage savings:

    $x(-h)+(1-x)(v-w-h) = x(-w) + (1-x)(v-w)$

    → $h = xw$ → $x= h/w$

	I	NI
S	0,-h	w,-w
W	w-g, v-w-h	w-g, v-w

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

- Another example: Battle of the sexes
- Man & Woman; Ballet or Football

	B	F
F	0, 0	2, 1
B	1, 2	0, 0

- Another example: Game of chicken
- Driver 1 & Driver 2; Tough or Weak

	T	W
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- Two pure NE: (F;F) and (B;B)
- One mixed NE: Indifference condition

→ Let $\sigma_1(F)=x$ and $\sigma_2(B)=y$ →
Player 1's indifference:
0 y + 2(1-y) = 1 y + 0 (1-y) → y=2/3
Player 2's indifference:
0 x + 2(1-x) = 1 x + 0 (1-x) → x=2/3
→ Mixed NE: ((2/3, 1/3); (2/3, 1/3))

	B	F
F	0, 0	2, 1
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- Another example: Game of chicken
- (same reasoning) →

Mixed NE: ((1/2, 1/2); (1/2, 1/2))

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Games in Strategic Form & Nash Equilibrium

Nash Equilibrium: More than one NE

Focal points

- Some games have more than one NE → which will be chosen?
- Theory of „focalness“ of NE („focal points“): Example: Chose time of day simultaneously; reward if match: 12 noon is focal, 15:37 is not

Risk Dominance

- Stag Hunt: NE: (C;C) and (D;D); (C;C) is pareto-dominant → (C;C) might be chosen if p(C)>0.5 BUT
- more than two players: ALL have to agree on C → $p(C)^8 > 0.5$ → p(C)>0.93 → (D;D) „risk dominates“ (C;C)

	Hunt Stag (C)	Hunt Hare (D)
Hunt Stag (C)	2 , 2	0 , 1
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