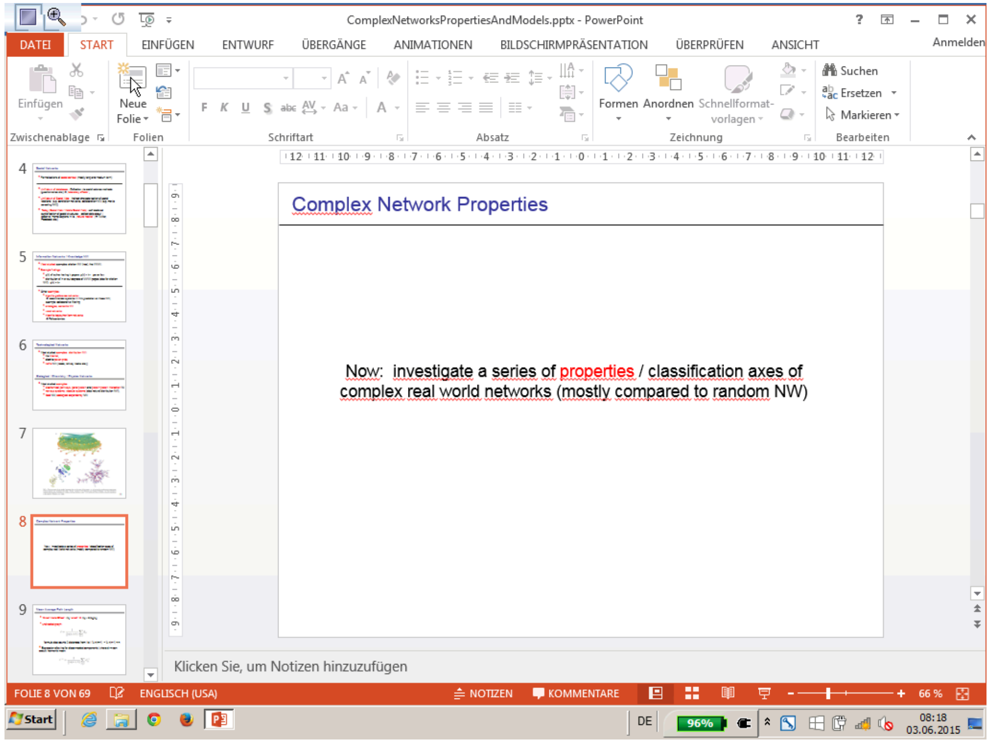
Script **generated by TTT**

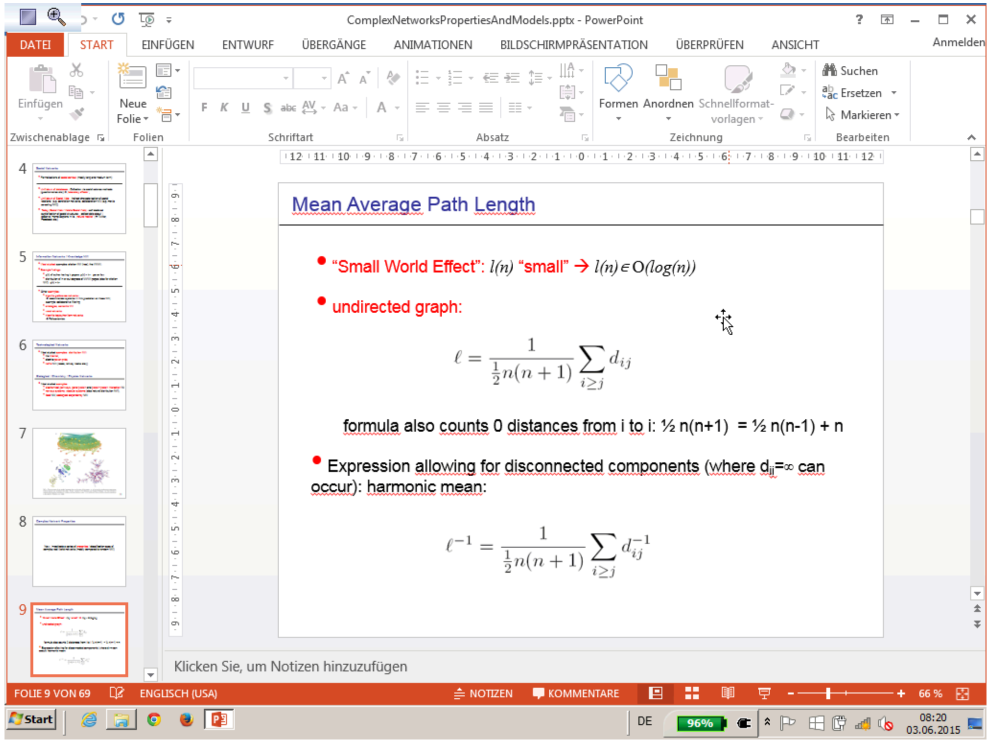
Title: groh: profile1 (03.06.2015)

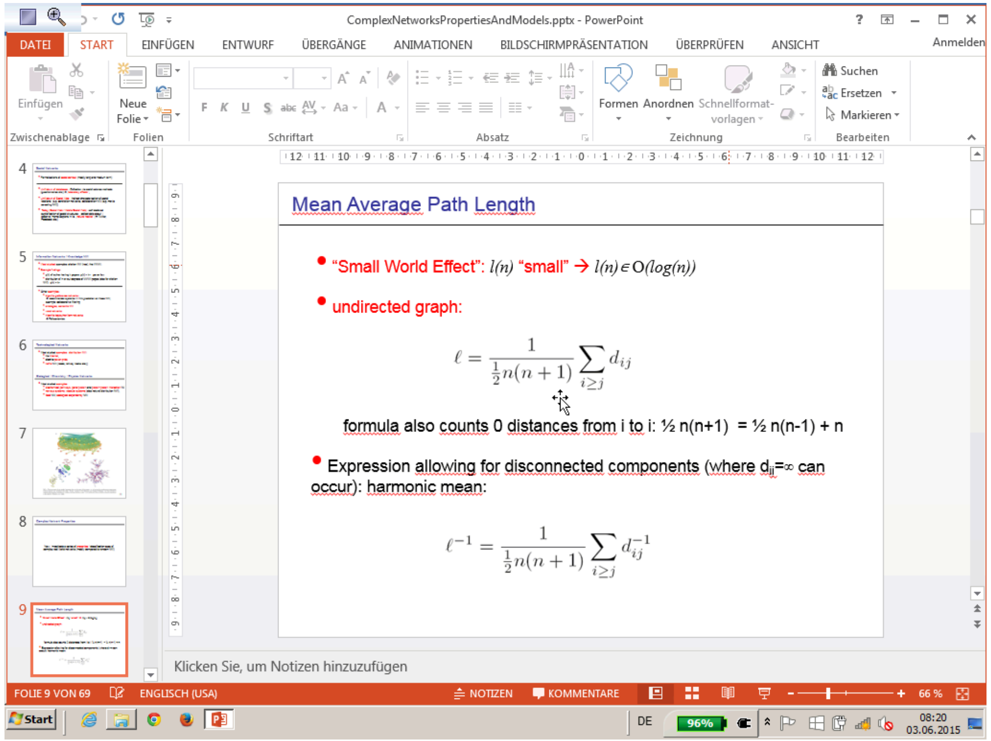
Date: Wed Jun 03 08:18:45 CEST 2015

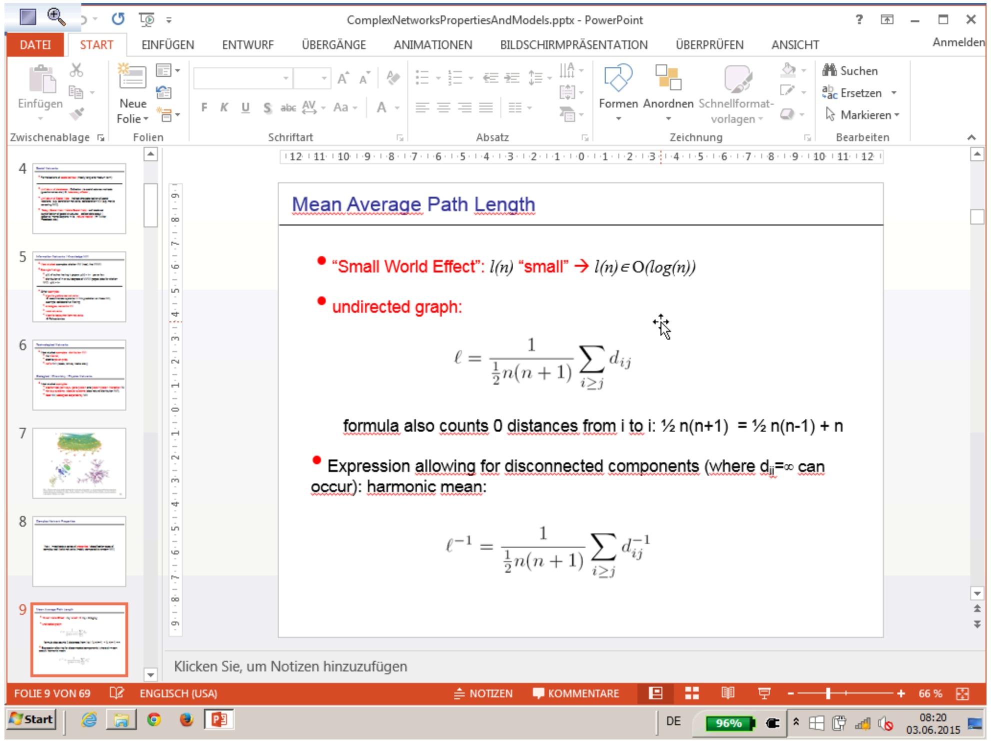
Duration: 85:53 min

Pages: 101









Transitivity / Clustering Coefficient

- Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{6\times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

p(FOAF)

- Clustering coefficient (Watts-Strogatz-version, for node i):

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i} = \frac{|\{e_{\{kj\}} \mid v_k, v_j \in N_i\}|}{\frac{k_i(k_i-1)}{2}}$$

(see Introduction , k_i = degree of node i)

Clustering coefficient (Watts-Strogatz-version, for whole graph):

$$C = C^{(2)} = \frac{1}{n}\sum_i C_i$$

mean of ratio instead of ratio of means

Transitivity / Clustering Coefficient

- Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{6\times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

p(FOAF)

- Clustering coefficient (Watts-Strogatz-version, for node i):

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i} = \frac{|\{e_{\{kj\}} \mid v_k, v_j \in N_i\}|}{\frac{k_i(k_i-1)}{2}}$$

(see Introduction , k_i = degree of node i)

Clustering coefficient (Watts-Strogatz-version, for whole graph):

$$C = C^{(2)} = \frac{1}{n}\sum_i C_i$$

mean of ratio instead of ratio of means

Transitivity / Clustering Coefficient

- Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{6\times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

p(FOAF)

- Clustering coefficient (Watts-Strogatz-version, for node i):

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i} = \frac{|\{e_{\{kj\}} \mid v_k, v_j \in N_i\}|}{\frac{k_i(k_i-1)}{2}}$$

(see Introduction , k_i = degree of node i)

Clustering coefficient (Watts-Strogatz-version, for whole graph):

$$C = C^{(2)} = \frac{1}{n}\sum_i C_i$$

mean of ratio instead of ratio of means

Transitivity / Clustering Coefficient

- Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{6\times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

p(FOAF)

- Clustering coefficient (Watts-Strogatz-version, for node i):

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i} = \frac{|\{e_{\{kj\}} \mid v_k, v_j \in N_i\}|}{\frac{k_i(k_i-1)}{2}}$$

(see Introduction , k_i = degree of node i)

Clustering coefficient (Watts-Strogatz-version, for whole graph):

$$C = C^{(2)} = \frac{1}{n}\sum_i C_i$$

mean of ratio instead of ratio of means

Transitivity / Clustering Coefficient

- Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{6\times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

p(FOAF)

- Clustering coefficient (Watts-Strogatz-version, for node i):

$$C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i} = \frac{|\{e_{\{kj\}} \mid v_k, v_j \in N_i\}|}{\frac{k_i(k_i-1)}{2}}$$

(see Introduction , k_i = degree of node i)

Clustering coefficient (Watts-Strogatz-version, for whole graph):

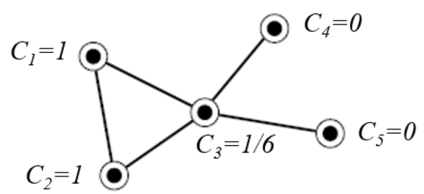
$$C = C^{(2)} = \frac{1}{n}\sum_i C_i$$

mean of ratio instead of ratio of means

Transitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3\times 1}{8} = 0.375$$



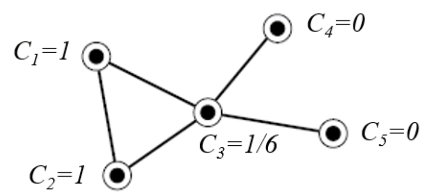
$$C^{(2)} = \frac{1}{n}\sum_i C_i \quad \text{with} \quad C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$C^{(2)}$ = 1/5 (1 + 1 + 1/6 + 0 + 0) = 13/30 = 0.433333

Transitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3\times 1}{8} = 0.375$$



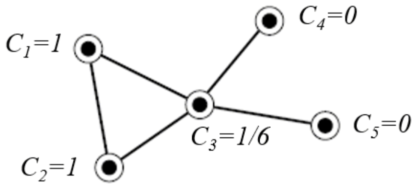
$$C^{(2)} = \frac{1}{n}\sum_i C_i \quad \text{with} \quad C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$C^{(2)}$ = 1/5 (1 + 1 + 1/6 + 0 + 0) = 13/30 = 0.433333

Transitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3\times 1}{8} = 0.375$$



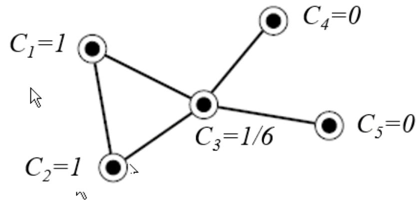
$$C^{(2)} = \frac{1}{n}\sum_i C_i \quad \text{with} \quad C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$C^{(2)} = 1/5\ (1 + 1 + 1/6 + 0 + 0) = 13/30 = 0.433333$

Transitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3\times 1}{8} = 0.375$$



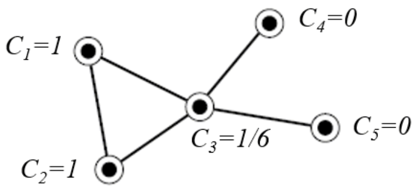
$$C^{(2)} = \frac{1}{n}\sum_i C_i \quad \text{with} \quad C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$C^{(2)} = 1/5\ (1 + 1 + 1/6 + 0 + 0) = 13/30 = 0.433333$

Transitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3\times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3\times 1}{8} = 0.375$$



$$C^{(2)} = \frac{1}{n}\sum_i C_i \quad \text{with} \quad C_i = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$$

$C^{(2)} = 1/5\ (1 + 1 + 1/6 + 0 + 0) = 13/30 = 0.433333$

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ıber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ıber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ıber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ıber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	–0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	–0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	–0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	–0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	–0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	–0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	–0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	–0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	–0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	–0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	–0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	–0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	–0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	–0.226	416, 421

BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Sec
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

Degree Distribution

- Notation:
  p(k) = p_k = fraction of nodes having degree k
- Cumulative distribution:
  $$P_k = \sum_{k'=k}^{\infty} p_{k'}$$
- power law:
  $$p_k \sim k^{-\alpha}$$
  $$\rightarrow \quad P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$
- exponential:
  $$p_k \sim e^{-k/\kappa}$$
  $$\rightarrow \quad P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

- Notation:
  p(k) = p_k = fraction of nodes having degree k
- Cumulative distribution:
  $$P_k = \sum_{k'=k}^{\infty} p_{k'}$$
- power law:
  $$p_k \sim k^{-\alpha}$$
  $$\rightarrow \quad P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$
- exponential:
  $$p_k \sim e^{-k/\kappa}$$
  $$\rightarrow \quad P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

- Notation:
  p(k) = p_k = fraction of nodes having degree k
- Cumulative distribution:
  $$P_k = \sum_{k'=k}^{\infty} p_{k'}$$
- power law:
  $$p_k \sim k^{-\alpha}$$
  $$\rightarrow \quad P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$
- exponential:
  $$p_k \sim e^{-k/\kappa}$$
  $$\rightarrow \quad P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

- Notation:

  p(k) = p_k = fraction of nodes having degree k

- Cumulative distribution:

$$P_k = \sum_{k'=k}^{\infty} p_{k'}$$

- power law:

$$p_k \sim k^{-\alpha}$$

$$\rightarrow \quad P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$

- exponential:

$$p_k \sim e^{-k/\kappa}$$

$$\rightarrow \quad P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

- Notation:

  p(k) = p_k = fraction of nodes having degree k

- Cumulative distribution:

$$P_k = \sum_{k'=k}^{\infty} p_{k'}$$

- power law:

$$p_k \sim k^{-\alpha}$$

$$\rightarrow \quad P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$

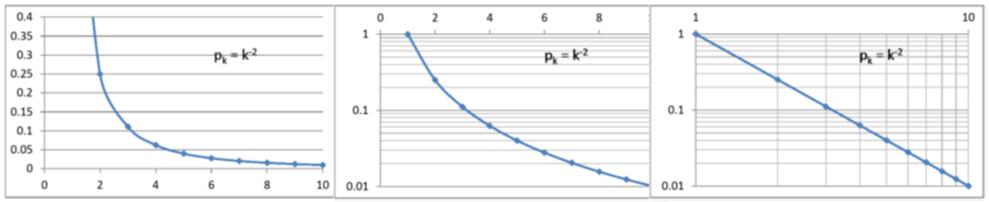
- exponential:

$$p_k \sim e^{-k/\kappa}$$

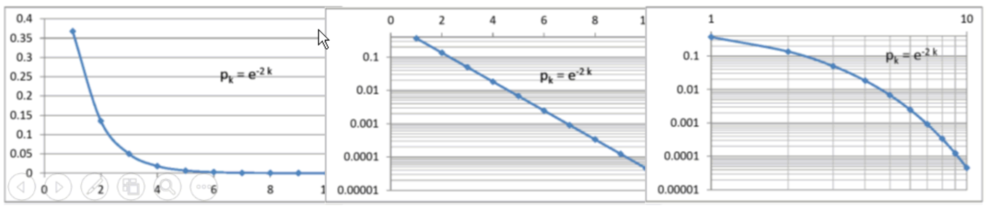
$$\rightarrow \quad P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

$$p_k \sim k^{-\alpha}$$

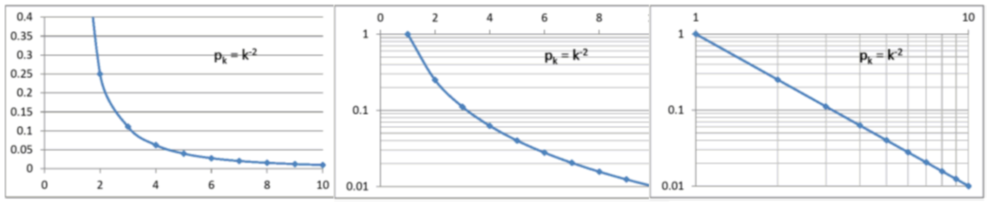


$$p_k \sim e^{-k/\kappa}$$

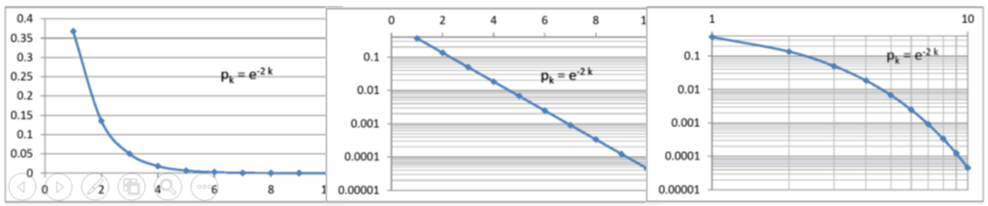


Degree Distribution

$$p_k \sim k^{-\alpha}$$

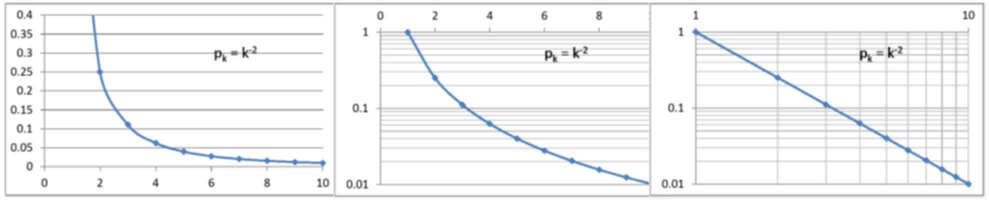


$$p_k \sim e^{-k/\kappa}$$

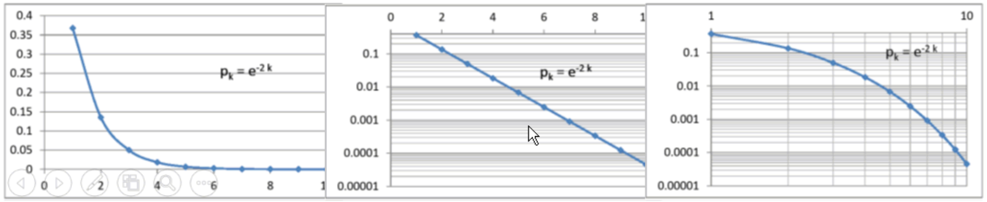


Degree Distribution

$$p_k \sim k^{-\alpha}$$

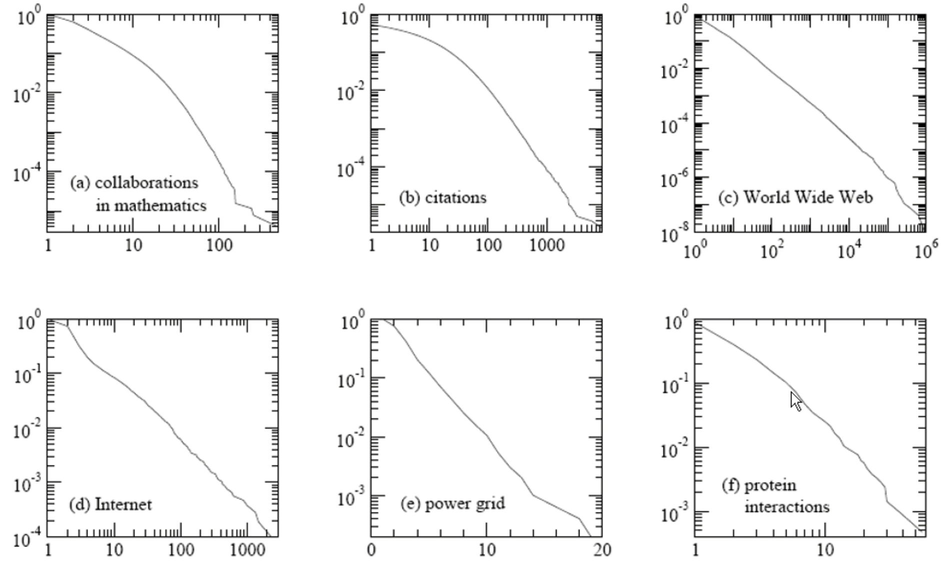


$$p_k \sim e^{-k/\kappa}$$



Degree Distribution

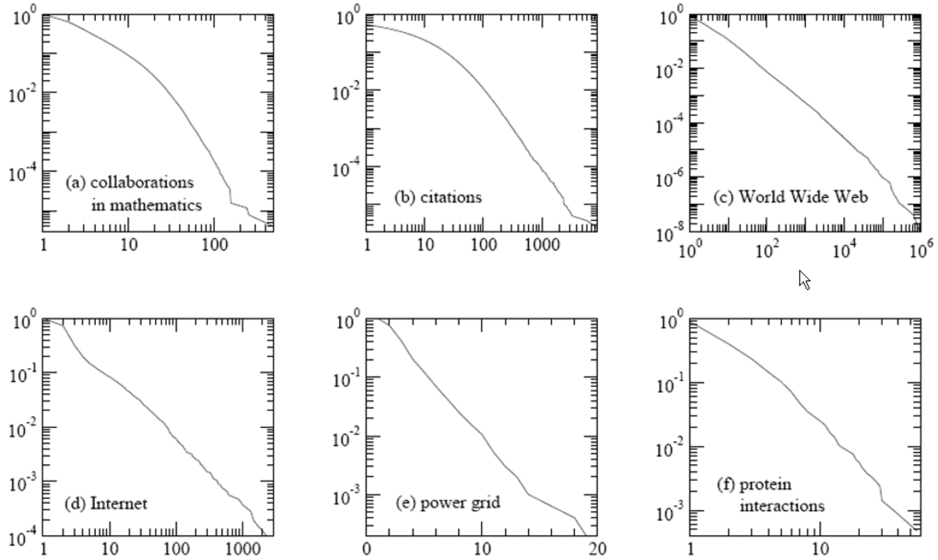
Cumulative distributions P_k of example real world NW



[1]

Degree Distribution

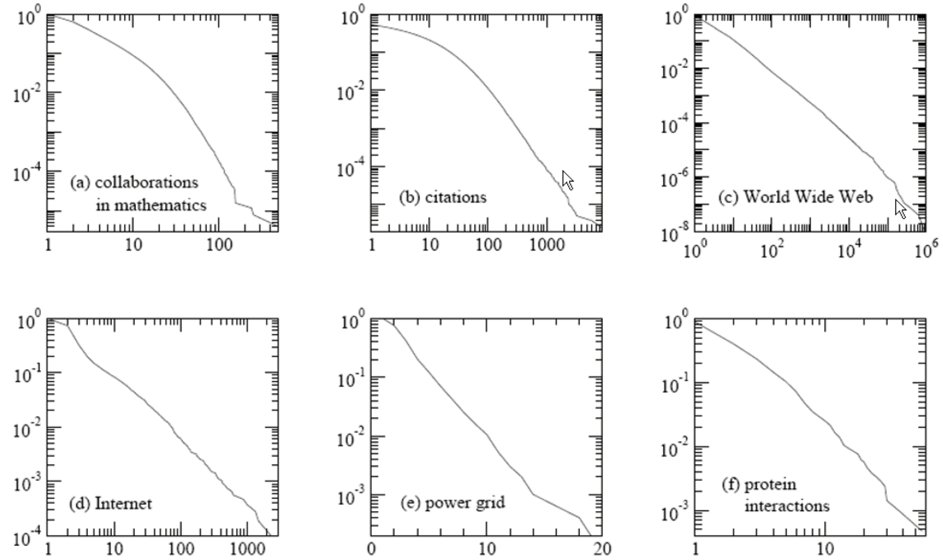
Cumulative distributions P_k of example real world NW



[1]

Degree Distribution

Cumulative distributions P_k of example real world NW



[1]

Degree Distribution

"Power law" == "Scale free":

- $f(x) = x^{\alpha}$ is only solution to functional equation formalizing scale freedom $f(ax) = b\, f(x)$
- in other words: change of scale → f still „looks the same"
- other point of view:

Although we can compute the expectation $E(k)=\Sigma_k\, k\, k^{-\alpha}$ if $\alpha>1$,
the variance (error bars) $Var(k)= \Sigma_k\, (k-E(k))^2\, k^{-\alpha}$
diverges → we „cannot be shure about k"
→ „no characteristic scale" → „scale free"

Degree Distribution

"Power law" == "Scale free":

- $f(x) = x^{\alpha}$ is only solution to functional equation formalizing scale freedom $f(ax) = b\, f(x)$
- in other words: change of scale → f still „looks the same"
- other point of view:

Although we can compute the expectation $E(k)=\Sigma_k\, k\, k^{-\alpha}$ if $\alpha>1$,
the variance (error bars) $Var(k)= \Sigma_k\, (k-E(k))^2\, k^{-\alpha}$
diverges → we „cannot be shure about k"
→ „no characteristic scale" → „scale free"

Degree Distribution

Examples:

- Power law: citation NW, WWW, Internet, metabolic NW, telephone call NW, human sexual contact NW etc.
- Exponential: power grid, railway NW
- Power law with exp. cut-offs: Movie co-actor NW

Degree Distribution

Examples:

- Power law: citation NW, WWW, Internet, metabolic NW, telephone call NW, human sexual contact NW etc.
- Exponential: power grid, railway NW
- Power law with exp. cut-offs: Movie co-actor NW

Degree Distribution

Examples:

- Power law: citation NW, WWW, Internet, metabolic NW, telephone call NW, human sexual contact NW etc.
- Exponential: power grid, railway NW
- Power law with exp. cut-offs: Movie co-actor NW

Maximum Degree

- „less or equal than one vertex with k_{max}“
→ $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
  $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
  $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
  $$k_{\max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
→ $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
  $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
  $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
  $$k_{\max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
→ $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
  $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
  $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
  $$k_{\max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1 - P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1 - P_k)^{n-m} = (p_k + 1 - P_k)^n - (1 - P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1 - P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1 - P_k)^{n-m} = (p_k + 1 - P_k)^n - (1 - P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1 - P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1 - P_k)^{n-m} = (p_k + 1 - P_k)^n - (1 - P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1 - P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1 - P_k)^{n-m} = (p_k + 1 - P_k)^n - (1 - P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- „less or equal than one vertex with k_{max}“
  → $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
  but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:
    $$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$
  - → prob of k being the highest degree in graph:
    $$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k + 1 - P_k)^n - (1-P_k)^n$$
  - → expected highest degree:
    $$k_{max} = \sum_k k h_k$$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk}h_k = n\left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k\right)(p_k+1-P_k)^{n-1} + p_k(1-P_k)^{n-1}\right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -np_k^2$$

(assuming: p_k is small for k > k_{max} and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk}h_k = n\left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k\right)(p_k+1-P_k)^{n-1} + p_k(1-P_k)^{n-1}\right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -np_k^2$$

(assuming: p_k is small for k > k_{max} and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- „less or equal than one vertex with k_{max}“
→ $np_{k\_max} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$
but: not very accurate estimation

- Other estimation:
  - prob p of „exactly m nodes with k and rest of nodes smaller than k“:

$$\binom{n}{m} p_k^m (1-P_k)^{n-m}$$

  - → prob of k being the highest degree in graph:

$$h_k = \sum_{m=1}^{n} \binom{n}{m} p_k^m (1-P_k)^{n-m} = (p_k+1-P_k)^n - (1-P_k)^n$$

  - → expected highest degree:

$$k_{\max} = \sum_k k h_k$$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk}h_k = n\left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k\right)(p_k+1-P_k)^{n-1} + p_k(1-P_k)^{n-1}\right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -np_k^2$$

(assuming: p_k is small for k > k_{max} and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk} h_k = n \left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k \right)(p_k + 1 - P_k)^{n-1} + p_k (1 - P_k)^{n-1} \right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -n p_k^2$$

(assuming: p_k is small for $k > k_{max}$ and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk} h_k = n \left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k \right)(p_k + 1 - P_k)^{n-1} + p_k (1 - P_k)^{n-1} \right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -n p_k^2$$

(assuming: p_k is small for $k > k_{max}$ and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk} h_k = n \left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k \right)(p_k + 1 - P_k)^{n-1} + p_k (1 - P_k)^{n-1} \right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -n p_k^2$$

(assuming: p_k is small for $k > k_{max}$ and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Maximum Degree

- since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

modal value : $\frac{d}{dk} h_k = 0$

Using $\mathrm{d}P_k/\mathrm{d}k = p_k$ we get

$$\frac{d}{dk} h_k = n \left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k} - p_k \right)(p_k + 1 - P_k)^{n-1} + p_k (1 - P_k)^{n-1} \right] = 0$$

or $k_{\max}$ is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -n p_k^2$$

(assuming: p_k is small for $k > k_{max}$ and that $np_k \ll 1$ and that $P_k \ll 1$)

→we get for power law $p_k \sim k^{-\alpha}$ that $k_{\max} \sim n^{1/(\alpha-1)}$

Network Resilience

- What happens if nodes are removed? (interesting e.g. for vaccination effects in disease spreading in human contact networks)

- For power law networks:

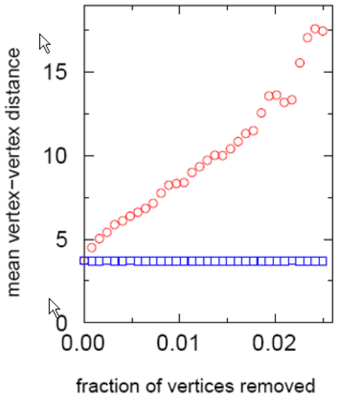
  remove random nodes :
  no effect on mean distances

  remove high degree nodes:
  drastic effect

- Interpretations:

  Internet is easy to attack

  Internet is not easy to attack



[1]

Network Resilience

- What happens if nodes are removed? (interesting e.g. for vaccination effects in disease spreading in human contact networks)

- For power law networks:

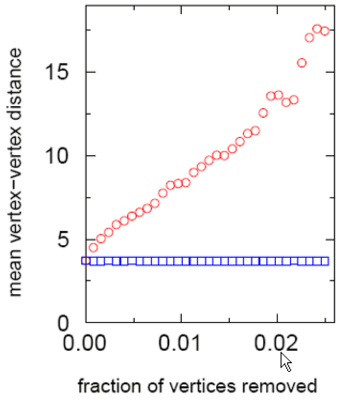
  remove random nodes :
  no effect on mean distances

  remove high degree nodes:
  drastic effect

- Interpretations:

  Internet is easy to attack

  Internet is not easy to attack



[1]

Network Resilience

- What happens if nodes are removed? (interesting e.g. for vaccination effects in disease spreading in human contact networks)

- For power law networks:

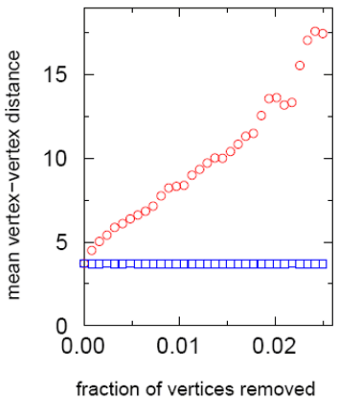
  remove random nodes :
  no effect on mean distances

  remove high degree nodes:
  drastic effect

- Interpretations:

  Internet is easy to attack

  Internet is not easy to attack



[1]

Mixing Patterns

- Ecological NW, Internet, some social NW:
  Assortative Mixing (Homophily): Nodes attach to similar nodes / nodes of same class OR
  Disassortative Mixing (Heterophily): Nodes attach to nodes of different classes (almost n-partite behavior)

- Diassortativity:
  Food Web: Plants ←→ Herbivores ←→ Carnivores
  but few Plants ←→ Plants etc.
  Internet: Backbones provider ←→ ISP ←→ end user
  but few ISP ←→ ISP etc.

- Assortativity:
  Social NW

Mixing Patterns

- Ecological NW, Internet, some social NW:
  Assortative Mixing (Homophily): Nodes attach to similar nodes / nodes of same class OR
  Disassortative Mixing (Heterophily): Nodes attach to nodes of different classes (almost n-partite behavior)

- Diassortativity:
  Food Web: Plants ←→ Herbivores ←→ Carnivores
  but few Plants ←→ Plants etc.
  Internet: Backbones provider ←→ ISP ←→ end user
  but few ISP ←→ ISP etc.

- Assortativity:
  Social NW

Mixing Patterns

- Ecological NW, Internet, some social NW:
  Assortative Mixing (Homophily): Nodes attach to similar nodes / nodes of same class OR
  Disassortative Mixing (Heterophily): Nodes attach to nodes of different classes (almost n-partite behavior)

- Diassortativity:
  Food Web: Plants ←→ Herbivores ←→ Carnivores
  but few Plants ←→ Plants etc.
  Internet: Backbones provider ←→ ISP ←→ end user
  but few ISP ←→ ISP etc.

- Assortativity:
  Social NW

Mixing Patterns

E =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathrm{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\sum_{ij} e_{ij} = 1,$ $\sum_j P(j|i) = 1$

Mixing Patterns

E =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathrm{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\sum_{ij} e_{ij} = 1,$ $\sum_j P(j|i) = 1$

Mixing Patterns

$\mathbf{E}$ =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathbf{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\quad \sum_{ij} e_{ij} = 1, \quad \sum_j P(j|i) = 1$

Mixing Patterns

$\mathbf{E}$ =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathbf{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\quad \sum_{ij} e_{ij} = 1, \quad \sum_j P(j|i) = 1$

Mixing Patterns

$\mathbf{E}$ =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathbf{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\quad \sum_{ij} e_{ij} = 1, \quad \sum_j P(j|i) = 1$

Mixing Patterns

$\mathbf{E}$ =

		women			
		black	hispanic	white	other
men	black	506	32	69	26
	hispanic	23	308	114	38
	white	26	46	599	68
	other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302]. [1]

- measure mixing: analogous to modularity: mixing matrix $\mathbf{e} = \frac{\mathbf{E}}{\|\mathbf{E}\|}$

→ $P(j|i) = e_{ij}/\sum_{j'} e_{ij'}$ $\quad \sum_{ij} e_{ij} = 1, \quad \sum_j P(j|i) = 1$

Mixing Patterns

E =

men \ women	black	hispanic	white	other
black	506	32	69	26
hispanic	23	308	114	38
white	26	46	599	68
other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302].

- → first measure for Assortativity:

$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

issues: Asymmetry of E → two values;
Not respecting size of classes

- → second measure for Assortativity: (cmp. Modularity)

$$r = \frac{\mathrm{Tr}\,\mathbf{e} - \|\,\mathbf{e}^2\,\|}{1 - \|\,\mathbf{e}^2\,\|}$$

Mixing Patterns

E =

men \ women	black	hispanic	white	other
black	506	32	69	26
hispanic	23	308	114	38
white	26	46	599	68
other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302].

- → first measure for Assortativity:

$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

issues: Asymmetry of E → two values;
Not respecting size of classes

- → second measure for Assortativity: (cmp. Modularity)

$$r = \frac{\mathrm{Tr}\,\mathbf{e} - \|\,\mathbf{e}^2\,\|}{1 - \|\,\mathbf{e}^2\,\|}$$

Mixing Patterns

E =

men \ women	black	hispanic	white	other
black	506	32	69	26
hispanic	23	308	114	38
white	26	46	599	68
other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302].

- → first measure for Assortativity:

$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

issues: Asymmetry of E → two values;
Not respecting size of classes

- → second measure for Assortativity: (cmp. Modularity)

$$r = \frac{\mathrm{Tr}\,\mathbf{e} - \|\,\mathbf{e}^2\,\|}{1 - \|\,\mathbf{e}^2\,\|}$$

Mixing Patterns

E =

men \ women	black	hispanic	white	other
black	506	32	69	26
hispanic	23	308	114	38
white	26	46	599	68
other	10	14	47	32

TABLE III Couples in the study of Catania *et al.* [85] tabulated by race of either partner. After Morris [302].

- → first measure for Assortativity:

$$Q = \frac{\sum_i P(i|i) - 1}{N - 1}$$

issues: Asymmetry of E → two values;
Not respecting size of classes

- → second measure for Assortativity: (cmp. Modularity)

$$r = \frac{\mathrm{Tr}\,\mathbf{e} - \|\,\mathbf{e}^2\,\|}{1 - \|\,\mathbf{e}^2\,\|}$$

Mixing Patterns

- Special example: „class“ of nodes determined by degree
  → nodes attached to nodes with same or different degree?
  Both variants occur in real world NW

- Degree correlation measures:

  1) mean degree of neighbors of node with degree k:
     → if assortative mixing: curve should be increasing
     → Internet: curve decreases → diassortativity

  2) Pearson correlation for node degrees k_i and k_j of adjacent nodes i and j

Mixing Patterns

- Special example: „class“ of nodes determined by degree
  → nodes attached to nodes with same or different degree?
  Both variants occur in real world NW

- Degree correlation measures:

  1) mean degree of neighbors of node with degree k:
     → if assortative mixing: curve should be increasing
     → Internet: curve decreases → diassortativity

  2) Pearson correlation for node degrees k_i and k_j of adjacent nodes i and j

Mixing Patterns

- Special example: „class“ of nodes determined by degree
  → nodes attached to nodes with same or different degree?
  Both variants occur in real world NW

- Degree correlation measures:

  1) mean degree of neighbors of node with degree k:
     → if assortative mixing: curve should be increasing
     → Internet: curve decreases → diassortativity

  2) Pearson correlation for node degrees k_i and k_j of adjacent nodes i and j

Mixing Patterns

- Special example: „class“ of nodes determined by degree
  → nodes attached to nodes with same or different degree?
  Both variants occur in real world NW

- Degree correlation measures:

  1) mean degree of neighbors of node with degree k:
     → if assortative mixing: curve should be increasing
     → Internet: curve decreases → diassortativity

  2) Pearson correlation for node degrees k_i and k_j of adjacent nodes i and j

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45	0.56	0.363	311, 313
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	–	0.005	0.001	−0.029	45
	sexual contacts	undirected	2 810				3.2				265, 266
information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	−0.067	14, 34
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
	citation network	directed	783 339	6 716 198	8.57		3.0/–				351
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	−0.189	86, 148
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10	0.080	−0.003	416
	train routes	undirected	587	19 603	66.79	2.16	–		0.69	−0.033	366
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070	0.082	−0.016	318
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033	0.012	−0.119	395
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	−0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	−0.366	6, 354
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	−0.240	214
	protein interactions	undirected	2 115	2 240	2.12	6.80	2.4	0.072	0.071	−0.156	212
	marine food web	directed	135	598	4.43	2.05	–	0.16	0.23	−0.263	204
	freshwater food web	directed	92	997	10.84	1.90	–	0.20	0.087	−0.326	272
	neural network	directed	307	2 359	7.68	3.97	–	0.18	0.28	−0.226	416, 421

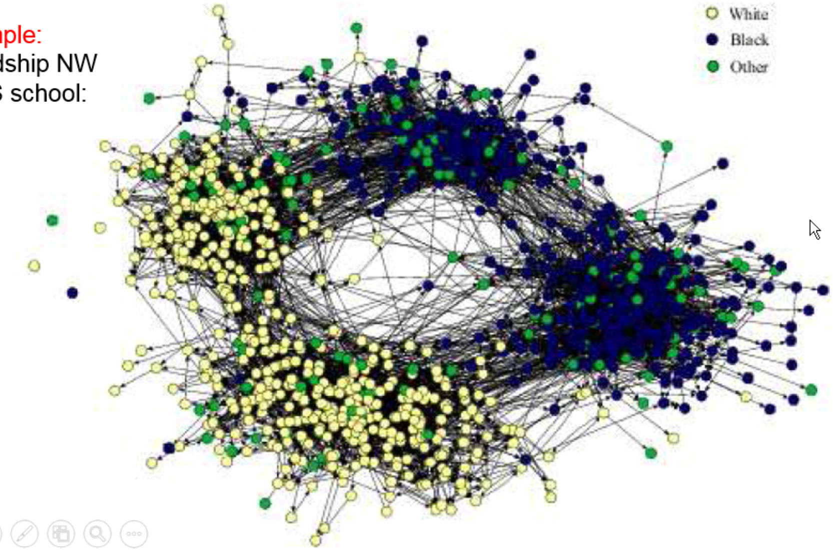
BLE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices
ber of edges m; mean degree z; mean vertex–vertex distance ℓ; exponent α of degree distribution if the distribution follows a power law (or "–" if not; in/out
onents are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Se
last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

[1]

Community and Group Structure

- Is NW well clustered? → see Parts on Clustering

example:
friendship NW
in US school:

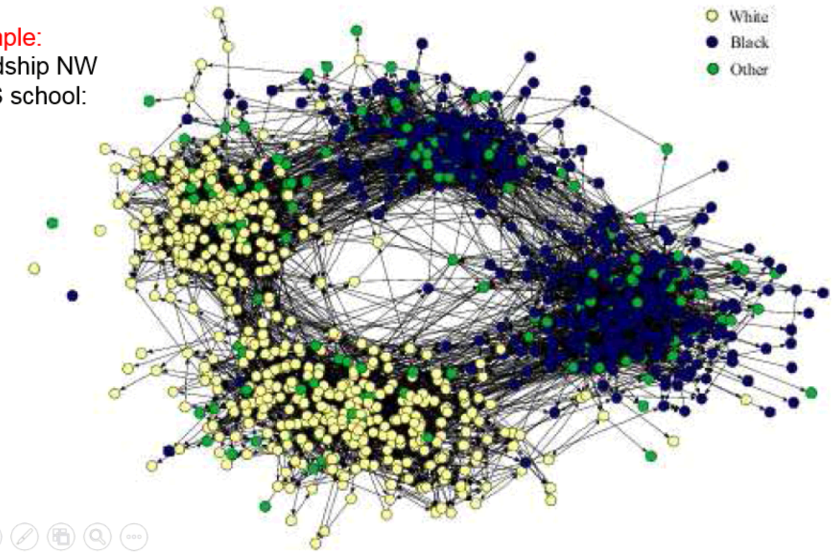


[1]

Community and Group Structure

- Is NW well clustered? → see Parts on Clustering

example:
friendship NW
in US school:



[1]

Navigatability of NW

- Milgram showed: short paths exist
  BUT: How do people find them?

→ see Part „Social Networks in Time and Space“

Component Structure

- Does a giant component exist?

→ see section on random graphs

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- $G_{n,p}$: space of graphs with n nodes and each of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed
(Poisson approximation of Binomial distribution)
→ „Poisson random graphs“

Random Graph Models: Poisson Graph

- Given: property Q („is connected", „has diameter xyz" etc.) of $G_{n,p}$:
  „$G_{n,p}$ has property Q with high probability": $P(Q|n,p) \to 1$ iff $n \to \infty$
  (adaptated from [2] (which, in turn, is adaptated from [3]))

- In such models $G_{n,p}$ phase transitions exist for properties Q:
  „threshold function" $q(n)$ (with $q(n) \to \infty$ if $n \to \infty$) so that:

$$\lim_{n\to\infty} P(Q|n,p) = \begin{cases} 0 & \text{if } \lim_{n\to\infty} p(n) / q(n) = 0 \\ 1 & \text{if } \lim_{n\to\infty} p(n) / q(n) = \infty \end{cases}$$

(adaptated from [3])

Random Graph Models: Poisson Graph

- Given: property Q („is connected", „has diameter xyz" etc.) of $G_{n,p}$:
  „$G_{n,p}$ has property Q with high probability": $P(Q|n,p) \to 1$ iff $n \to \infty$
  (adaptated from [2] (which, in turn, is adaptated from [3]))

- In such models $G_{n,p}$ phase transitions exist for properties Q:
  „threshold function" $q(n)$ (with $q(n) \to \infty$ if $n \to \infty$) so that:

$$\lim_{n\to\infty} P(Q|n,p) = \begin{cases} 0 & \text{if } \lim_{n\to\infty} p(n) / q(n) = 0 \\ 1 & \text{if } \lim_{n\to\infty} p(n) / q(n) = \infty \end{cases}$$

(adaptated from [3])

Random Graph Models: Poisson Graph

- Given: property Q („is connected", „has diameter xyz" etc.) of $G_{n,p}$:
  „$G_{n,p}$ has property Q with high probability": $P(Q|n,p) \to 1$ iff $n \to \infty$
  (adaptated from [2] (which, in turn, is adaptated from [3]))

- In such models $G_{n,p}$ phase transitions exist for properties Q:
  „threshold function" $q(n)$ (with $q(n) \to \infty$ if $n \to \infty$) so that:

$$\lim_{n\to\infty} P(Q|n,p) = \begin{cases} 0 & \text{if } \lim_{n\to\infty} p(n) / q(n) = 0 \\ 1 & \text{if } \lim_{n\to\infty} p(n) / q(n) = \infty \end{cases}$$

(adaptated from [3])

Random Graph Models: Poisson Graph

- Given: property Q („is connected", „has diameter xyz" etc.) of $G_{n,p}$:
  „$G_{n,p}$ has property Q with high probability": $P(Q|n,p) \to 1$ iff $n \to \infty$
  (adaptated from [2] (which, in turn, is adaptated from [3]))

- In such models $G_{n,p}$ phase transitions exist for properties Q:
  „threshold function" $q(n)$ (with $q(n) \to \infty$ if $n \to \infty$) so that:

$$\lim_{n\to\infty} P(Q|n,p) = \begin{cases} 0 & \text{if } \lim_{n\to\infty} p(n) / q(n) = 0 \\ 1 & \text{if } \lim_{n\to\infty} p(n) / q(n) = \infty \end{cases}$$

(adaptated from [3])

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = e^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = e^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - e^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = e^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = e^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - e^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = e^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = e^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - e^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = e^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = e^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - e^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →
  $$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →
  $$S = 1 - \mathrm{e}^{-zS}$$

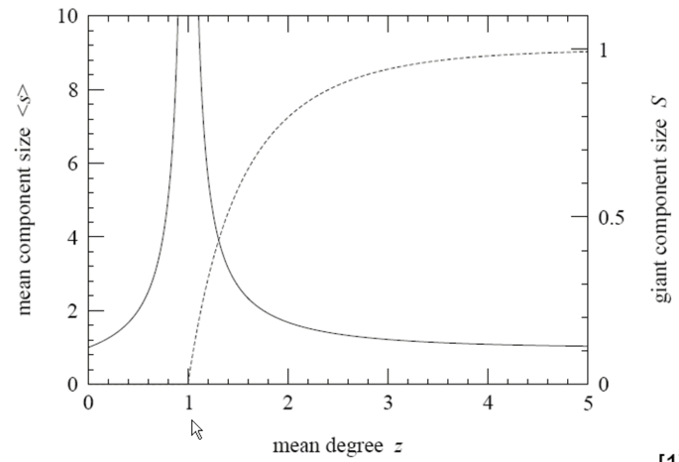
Random Graph Models: Poisson Graph

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →
  $$S = 1 - \mathrm{e}^{-zS}$$

Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$



[1]

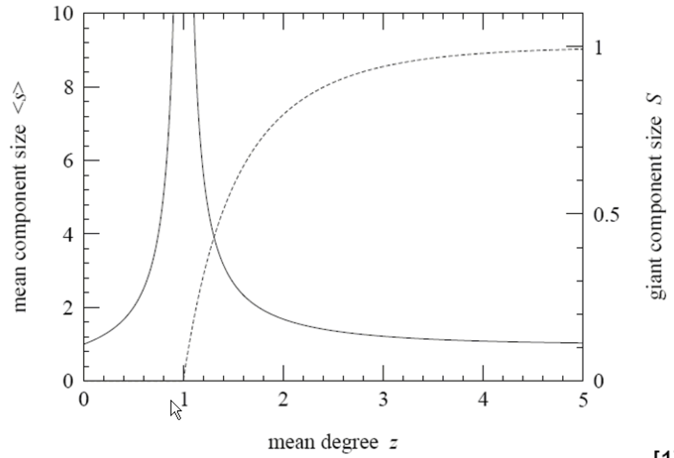
→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

Example: giant component / connectedness of $G_{n,p}$

- Let u be the fraction of nodes that do not belong to giant component X
  == probability for a given node i to be not in X
- probability for a given node i (with assumed degree k) to be not in X
  == probability that none of its neighbors is in X
  == u^k
- → u (k fixed) == u^k → $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- → fraction S of graph occupied by X is $S = 1 - u$ →

$$S = 1 - \mathrm{e}^{-zS}$$

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$

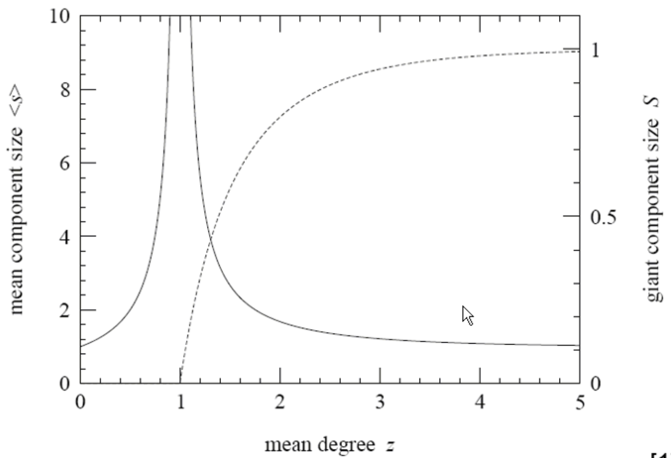


[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$

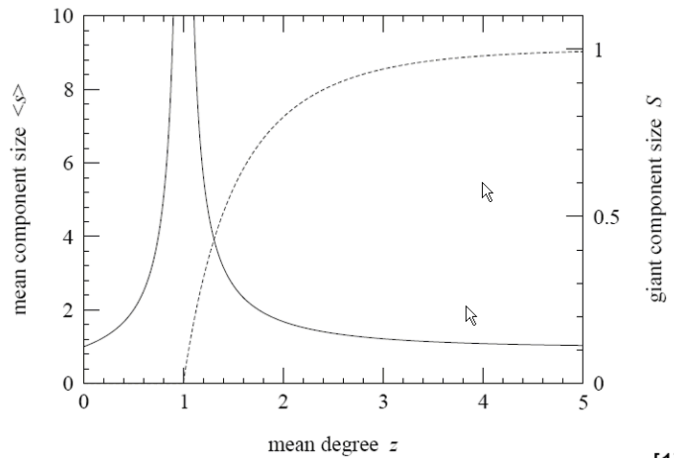


[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

Random Graph Models: Poisson Graph

- $S = 1 - \mathrm{e}^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$

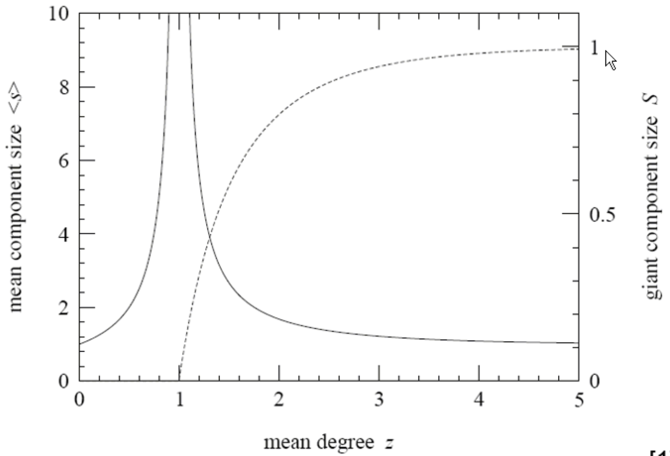


[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

Random Graph Models: Poisson Graph

- $S = 1 - e^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$

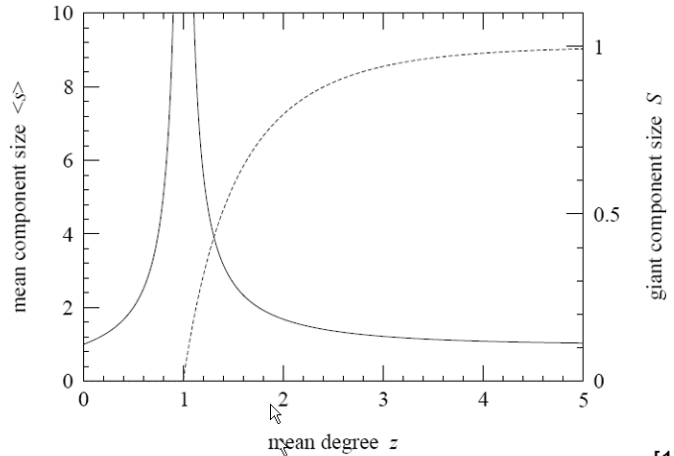


[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists

Random Graph Models: Poisson Graph

- $S = 1 - e^{-zS}$
- mean size <s> of smaller rest components (no proof): $\langle s \rangle = \frac{1}{1 - z + zS}$



[1]

→ if the av degree z is larger than 1 (== if p ~ (1+ε)/n): X exists