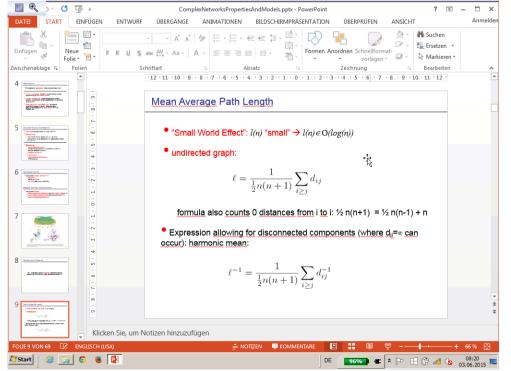
Script generated by TTT

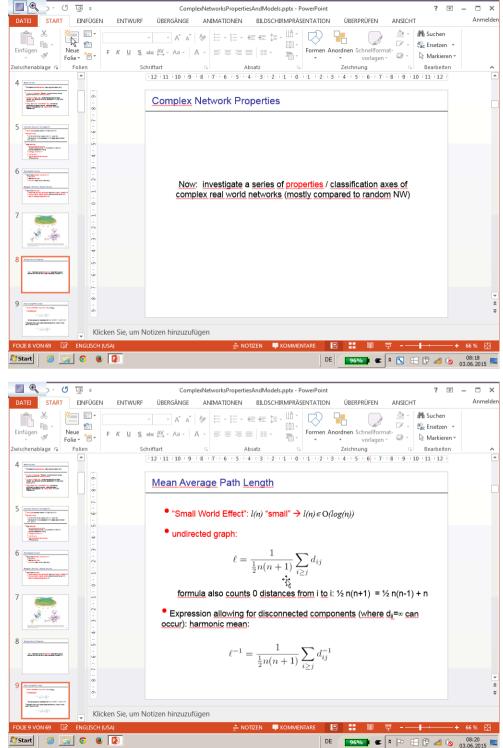
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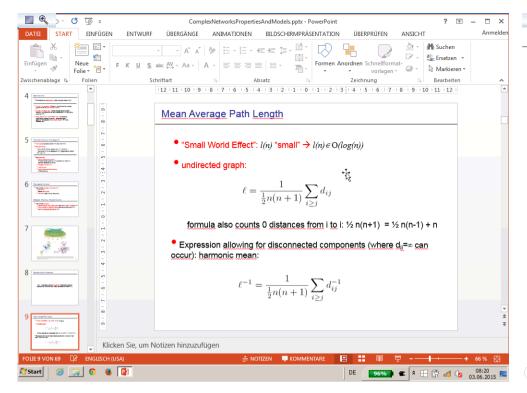
Date: Wed Jun 03 08:18:45 CEST 2015

Duration: 85:53 min

Pages: 101







ransitivity / Clustering Coefficient

Clustering coefficient (whole graph):

$$C = C^{(1)} = \frac{3 \times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}}$$
$$= \frac{6 \times \text{ number of triangles in the network}}{\text{number of paths of length two}}$$

Clustering coefficient (Watts-Strogatz-version, for node i):

$$\begin{split} C_i &= \frac{\text{number of triangles connected to vertex } \mathit{i}}{\text{number of triples centered on vertex } \mathit{i}} \\ &= \frac{|\left\{e_{\{kj\}} \mid v_k, v_j \in N_i\right\}|}{\underbrace{\frac{k_i(k_i - 1)}{2}}} \end{split} \tag{see Introduction }, k_i = \text{degree of node i}) \end{split}$$

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$$C = C^{(2)} = \frac{1}{n} \sum_{i} C_{i}$$

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mean of ratio instead of ratio of means

ransitivity / Clustering Coefficient

• Clustering coefficient (whole graph):

p(FOAF)

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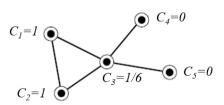
$$C = C^{(2)} = \frac{1}{n} \sum_{i} C_{i}$$

mean of ratio instead of ratio of means

ransitivity / Clustering Coefficient

Example:

$$C^{(1)} = \frac{3 \times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3 \times 1}{8} = \frac{0.375}{8}$$



$$C^{(2)} = \frac{1}{n} \sum_{i} C_{i}$$
 with $C_{i} = \frac{\text{number of triangles connected to vertex } i}{\text{number of triples centered on vertex } i}$

$$C^{(2)} = 1/5 (1 + 1 + 1/6 + 0 + 0) = 13/30^{10} = 0.433333$$

ransitivity / Clustering Coefficient

Clustering coefficient (whole graph):

$$C = C^{(I)} = \frac{3 \times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}}$$

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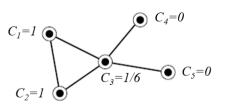
$$C = C^{(2)} = \frac{1}{n} \sum_{i} C_{i}$$

mean of ratio instead of ratio of means

ransitivity / Clustering Coefficient

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$$C^{(1)} = \frac{3 \times \text{ number of triangles in the network}}{\text{number of connected triples of vertices}} = \frac{3 \times 1}{8} = 0.375$$



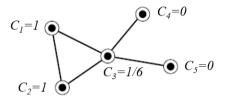
$$C^{(2)} = \frac{1}{n} \sum_{i} C_{i}$$
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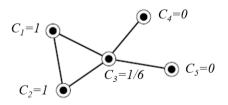
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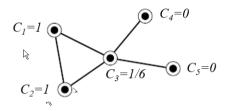
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	network	type	n	m	z	l	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
	film actors	undirected	449913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7673	55 392	14.44	4.60	-	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496489	3.92	7.57	-	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52909	245 300	9.27	6.19	-	0.45	0.56	0.363	311, 313
socia]	biology coauthorship	undirected	1 520 251	11803064	15.53	4.92	-	0.088	0.60	0.127	311, 313
90	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	599128	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16881	57 029	3.38	5.22	-	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	-	0.005	0.001	-0.029	45
	sexual contacts	undirected	2810				3.2				265, 266
п	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	14, 34
iti.	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7				74
information	citation network	directed	783 339	6716198	8.57		3.0/-				351
- Si	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	-	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	-0.189	86, 148
刁	power grid	undirected	4941	6594	2.67	18.99	-	0.10	0.080	-0.003	416
.02	train routes	undirected	587	19 603	66.79	2.16	-		0.69	-0.033	366
technological	software packages	directed	1 439	1723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
- g	software classes	directed	1 377	2 213	1.61	1.51	-	0.033	0.012	-0.119	395
ž	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	-0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	-0.366	6, 354
	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	-0.240	214
biological	protein interactions	undirected	2115	2 240	2.12	6.80	2.4	0.072	0.071	-0.156	212
ol l	marine food web	directed	135	598	4.43	2.05	-	0.16	0.23	-0.263	204
[oid	freshwater food web	directed	92	997	10.84	1.90	-	0.20	0.087	-0.326	272
	neural network	directed	307	2 359	7.68	3.97	-	0.18	0.28	-0.226	416, 421

3LE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices i ber of edges m; mean degree z; mean vertex—vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or "..." if not; in/out ments are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient c last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

	(A)

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social	biology coauthorship	undirected	1 520 251	11803064	15.53	4.92	-	0.088	0.60	0.127	311, 313
800	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16881	57 029	3.38	5.22	_	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	-	0.005	0.001	-0.029	45
	sexual contacts	undirected	2810				3.2				265, 266
	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	14, 34
tio.	WWW Altavista	directed	203 549 046	2130000000	10.46	16.18	2.1/2.7				74
Ë	citation network	directed	783 339	6716198	8.57		3.0/-				351
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18	power grid	undirected	4941	6594	2.67	18.99	-	0.10	0.080	-0.003	416
gi.	train routes	undirected	587	19 603	66.79	2.16	-		0.69	-0.033	366
technological	software packages	directed	1 439	1723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
큥	software classes	directed	1 377	2 213	1.61	1.51	_	0.033	0.012	-0.119	395
*	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	-0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	-0.366	6, 354
	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	-0.240	214
oiological	protein interactions	undirected	2115	2 240	2.12	6.80	2.4	0.072	0.071	-0.156	212
log	marine food web	directed	135	598	4.43	2.05	-	0.16	0.23	-0.263	204
biol	freshwater food web	directed	92	997	10.84	1.90	-	0.20	0.087	-0.326	272
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80	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59912	86 300	1.44	4.95	1.5/2.0		0.16		136
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7	power grid	undirected	4941	6594	2.67	18.99	-	0.10	0.080	-0.003	416
.56	train routes	undirected	587	19 603	66.79	2.16	-		0.69	-0.033	366
ď	software packages	directed	1 439	1723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
technological	software classes	directed	1 377	2 213	1.61	1.51	-	0.033	0.012	-0.119	395
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200	marine food web	directed	135	598	4.43	2.05	-	0.16	0.23	-0.263	204
-64	freshwater food web	directed	92	997	10.84	1.90	-	0.20	0.087	-0.326	272
	neural network	directed	307	2 359	7.68	3.97	-	0.18	0.28	-0.226	416, 421

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E	₩.										
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	email address books	directed	16881	57 029	3.38	5.22	-	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	-	0.005	0.001	-0.029	45
	sexual contacts	undirected	2810				3.2				265, 266
п	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	14, 34
-igi	WWW Altavista	directed	203 549 046	2130000000	10.46	16.18	2.1/2.7				74
information	citation network	directed	783 339	6716198	8.57		3.0/-				351
uţo	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	-	0.13	0.15	0.157	244
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	-0.189	86, 148
-	power grid	undirected	4941	6594	2.67	18.99	-	0.10	0.080	-0.003	416
-56	train routes	undirected	587	19 603	66.79	2.16	-		0.69	-0.033	366
or You	software packages	directed	1 439	1723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
technological	software classes	directed	1 377	2 213	1.61	1.51	-	0.033	0.012	-0.119	395
ŭ	electronic circuits	undirected	24097	53 248	4.34	11.05	3.0	0.010	0.030	-0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	-0.366	6, 354
	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	-0.240	214
biological	protein interactions	undirected	2115	2 240	2.12	6.80	2.4	0.072	0.071	-0.156	212
go	marine food web	directed	135	598	4.43	2.05	-	0.16	0.23	-0.263	204
bio	freshwater food web	directed	92	997	10.84	1.90	-	0.20	0.087	-0.326	272
	neural network	directed	307	2 359	7.68	3.97	-	0.18	0.28	-0.226	416, 421

3LE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices ι ber of edges m; mean degree z; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or "-" if not; in/out ments are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient $C^{(2)}$ from Eq. (6); and degree correlation coefficient r, Second last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.





⊕

	network	type	n	m	z	ℓ	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
	film actors	undirected	449913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7673	55 392	14.44	4.60	-	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496489	3.92	7.57	-	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52909	245 300	9.27	6.19	-	0.45	0.56	0.363	311, 313
social	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	-	0.088	0.60	0.127	311, 313
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59912	86 300	1.44	4.95	1.5/2.0		0.16		136
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Degree Distribution

• Notation:

 $p(k) = p_k = fraction of nodes having degree k$

Cumulative distribution:

$$P_k = \sum_{k'=k}^{\infty} p_{k'}$$

• power law:

$$p_k \sim k^{-\alpha}$$
 $\Rightarrow P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$

exponential:

$$p_k \sim e^{-k/\kappa}$$

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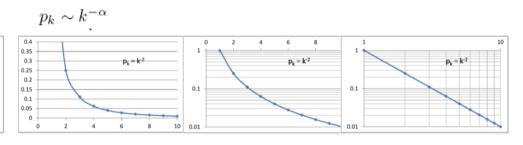
$$P_k \sim \sum_{k'=k}^{\infty} k'^{-\alpha} \sim k^{-(\alpha-1)}$$

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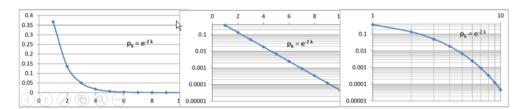
$$P_k = \sum_{k'=k}^{\infty} p_k \sim \sum_{k'=k}^{\infty} e^{-k'/\kappa} \sim e^{-k/\kappa}$$

Degree Distribution

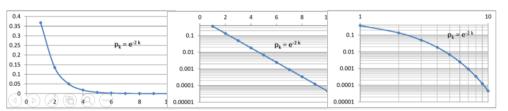


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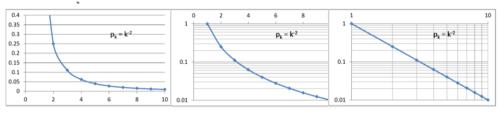
$$p_k \sim \mathrm{e}^{-k/\kappa}$$



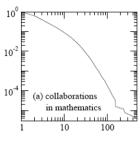
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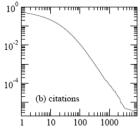
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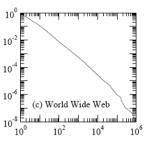


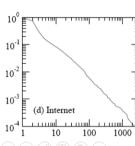


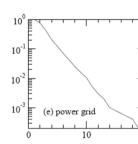
Cumulative distributions Pk of example real world NW

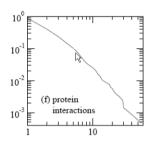








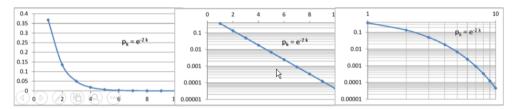




[1]

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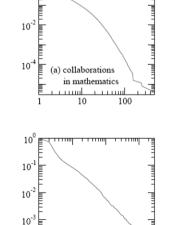


Degree Distribution

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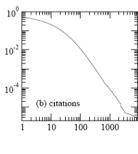
10° ≡

Cumulative distributions P_k of example real world NW



(d) Internet

10-4 1 10 100 1000



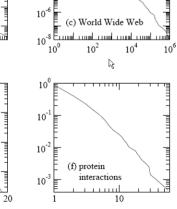
(e) power grid

10

10

10⁻¹ |

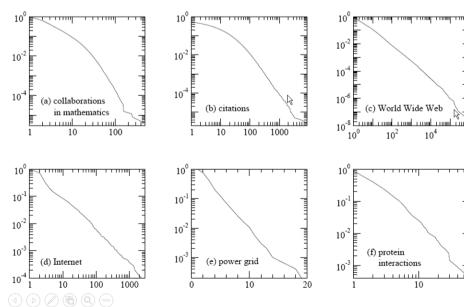
10⁻²



10⁻²

10-4

Cumulative distributions \boldsymbol{P}_k of example real world NW



Degree Distribution

Degree Distribution

"Power law" == "Scale free":

• $f(x) = x^{\alpha}$ is only solution to functional equation formalizing scale freedom f(ax) = b f(x)

• in other words: change of scale → f still "looks the same"

• other point of view.

Although we can compute the expectation $E(k)=\sum_k k \ k^{-\alpha}$ if $\alpha>1$, the variance (error bars) $Var(k)=\sum_k (k-E(k))^2 \ k^{-\alpha}$ diverges \rightarrow we "cannot be shure about k" \rightarrow "no characteristic scale" \rightarrow "scale free"

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Examples:

Power law: citation NW, WWW, Internet, metabolic NW, telephone call NW, human sexual contact NW etc.

Exponential: power grid, railway NW

Power law with exp. cut-offs: Movie co-actor NW



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ıvıaximum Degree

- "less or equal than one vertex with k_{max} "

 → $np_{k_{max}} = 1$ → for power law $p_k = k^{-\alpha}$: $k_{max} \sim n^{1/\alpha}$ but: $n = k^{-\alpha}$ very accurate estimation
- Other estimation:
 - prob p of "exactly m nodes with k and rest of nodes smaller than k":

$$\binom{n}{m}p_k^m(1-P_k)^{n-m}$$

◆ prob of k being the highest degree in graph:

$$h_k = \sum_{m=1}^n \binom{n}{m} p_k^m (1 - P_k)^{n-m}$$

= $(p_k + 1 - P_k)^n - (1 - P_k)^n$

expected highest degree:

$$k_{\text{max}} = \sum_{k} k h_k$$





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modal value :
$$\frac{d}{dk} h_k = 0$$

Using $dP_k/dk = p_k$ we get

$$\frac{d}{dk} h_k = n \left[\left(\frac{\mathrm{d}p_k}{\mathrm{d}k_k} - p_k \right) (p_k + 1 - P_k)^{n-1} + p_k (1 - P_k)^{n-1} \right] = 0$$

or k_{max} is a solution of

$$\frac{\mathrm{d}p_k}{\mathrm{d}k} \simeq -np_k^2$$

(assuming: $p_{\rm k}$ is small for k > k_{max} and that $np_k \ll 1$ and that $P_k \ll 1$)

 \rightarrow we get for power law $\,p_k \sim k^{-\alpha}\,\,$ that $\,\,k_{
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since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

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ıvıaximum Degree

since h_k is small for small k and also for large k → take as k_{max} the modal value of h_k →

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• What happens if nodes are removed? (interesting e.g. for vaccination effects in disease spreading in human contact networks)

R

For power law networks:

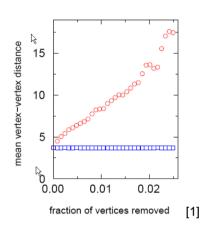
remove random nodes : no effect on mean distances

remove high degree nodes: drastic effect

Interpretations:

Internet is easy to attack

Internet is not easy to attack







Network Resilience

- What happens if nodes are removed? (interesting e.g. for vaccination effects in disease spreading in human contact networks)
- For power law networks:

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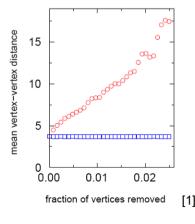
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19

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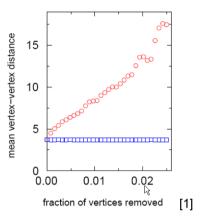
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iviixing Patterns

- Diassortativity:

Food Web: Plants ←→ Herbivores ←→ Carnivores but few Plants ←→ Plants etc.

Internet: Backbones provider $\leftarrow \rightarrow$ ISP $\leftarrow \rightarrow$ end user but few ISP $\leftarrow \rightarrow$ ISP etc.

• Assortativity:

Social NW





IVIIXING Patterns

Ecological NW, Internet, some social NW:

Assortative Mixing (Homophily): Nodes attach to similar nodes / nodes of same class OR

Disassortative Mixing (Heterophily): Nodes attach to nodes of different classes (almost n-partite behavior)

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$\mathbf{E} =$		black	506	32	69	26
	men	hispanic	23 18	308	114	38
	Ü	white	26	46	599	68
		other	10	14	47	32

TABLE III Couples in the study of Catania et al. [85] tabulated by race of either partner. After Morris [302].

ullet measure mixing: analogous to modularity: mixing matrix $ullet e = rac{\mathbf{E}}{\|\mathbf{E}\|}$

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first measure for Assortativity:

$$Q = \frac{\sum_{i} P(i|i) - 1}{N - 1}$$

issues: Asymmetry of E → two values; Not respecting size of classes

second measure for Assortativity: (cmp. Modularity)

$$r = \frac{\text{Tr } e - \|e^2\|}{1 - \|e^2\|}$$



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- Special example: "class" of nodes determined by degree
- → nodes attached to nodes with same or different degree?

 Both variants occur in real world NW
- Degree correlation measures:
 - 1) mean degree of neighbors of node with degree k:
 - → if assortative mixing: curve should be increasing
 - → Internet: curve decreases → diassortativity
 - 2) Pearson correlation for node degrees k_i and k_j of adjacent nodes i and j

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iviixing Patterns

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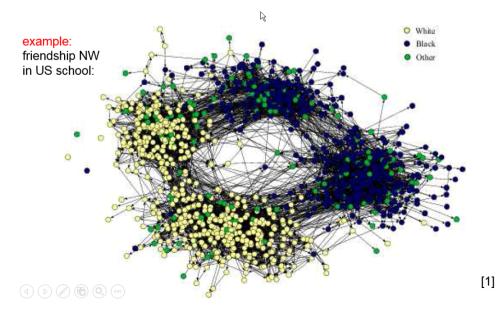


	network	type	n	m	z	l	α	$C^{(1)}$	$C^{(2)}$	r	Ref(s).
	film actors	undirected	449913	25 516 482	113.43	3.48	2.3	0.20	0.78	0.208	20, 416
	company directors	undirected	7673	55 392	14.44	4.60	-	0.59	0.88	0.276	105, 323
	math coauthorship	undirected	253 339	496489	3.92	7.57	-	0.15	0.34	0.120	107, 182
	physics coauthorship	undirected	52909	245 300	9.27	6.19	-	0.45	0.56	0.363	311, 313
social	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	-	0.088	0.60	0.127	311, 313
800	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1				8, 9
	email messages	directed	59912	86 300	1.44	4.95	1.5/2.0		0.16		136
	email address books	directed	16881	57 029	3.38	5.22	-	0.17	0.13	0.092	321
	student relationships	undirected	573	477	1.66	16.01	-	0.005	0.001	-0.029	45
	sexual contacts	undirected	2810				3.2				265, 266
п	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11	0.29	-0.067	14, 34
tio	WWW Altavista	directed	203 549 046	2130000000	10.46	16.18	2.1/2.7				74
information	citation network	directed	783 339	6716198	8.57		3.0/-				351
oju	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	-	0.13	0.15	0.157	244
.=	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7		0.44		119, 157
	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035	0.39	-0.189	86, 148
-6	power grid	undirected	4941	6 5 9 4	2.67	18.99	-	0.10	0.080	-0.003	416
ògi.	train routes	undirected	587	19 603	66.79	2.16	-		0.69	-0.033	366
technological	software packages	directed	1 439	1723	1.20	2.42	1.6/1.4	0.070	0.082	-0.016	318
- I	software classes	directed	1 377	2 213	1.61	1.51	-	0.033	0.012	-0.119	395
ž	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010	0.030	-0.154	155
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012	0.011	-0.366	6, 354
	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090	0.67	-0.240	214
biological	protein interactions	undirected	2115	2 240	2.12	6.80	2.4	0.072	0.071	-0.156	212
log	marine food web	directed	135	598	4.43	2.05	-	0.16	0.23	-0.263	204
bio	freshwater food web	directed	92	997	10.84	1.90	-	0.20	0.087	-0.326	272
	neural network	directed	307	2 359	7.68	3.97	-	0.18	0.28	-0.226	416, 421

3LE II Basic statistics for a number of published networks. The properties measured are: type of graph, directed or undirected; total number of vertices i ber of edges m; mean degree z; mean vertex-vertex distance ℓ ; exponent α of degree distribution if the distribution follows a power law (or "-" if not; in/out ments are given for directed graphs); clustering coefficient $C^{(1)}$ from Eq. (3); clustering coefficient power from Eq. (6); and degree correlation coefficient r, Set last column gives the citation(s) for the network in the bibliography. Blank entries indicate unavailable data.

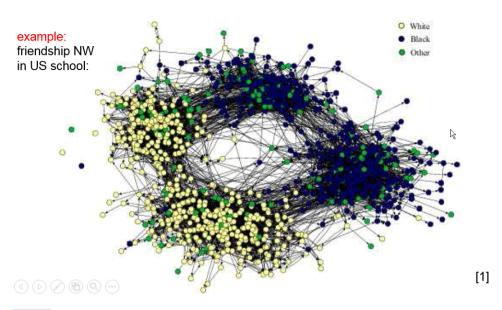
Community and Group Structure

Is NW well clustered? → see Parts on Clustering



Community and Group Structure

• Is NW well clustered? → see Parts on Clustering



Navigatability of NW

- Milgram showed: short paths exist BUT: How do people find them?
- → see Part "Social Networks in Time and Space"

Component Structure

- Does a giant component exist?
- → see section on random graphs

Random Graph Models: Poisson Graph

- G_{n,p}: space of graphs with n nodes and
 ach of the ½ n(n-1) edges appears with probability p
- p_k: probability that a node has degree k:

$$p_k = \binom{n}{k} p^k (1-p)^{n-k} \simeq \frac{z^k e^{-z}}{k!}$$

for n → ∞ and holding the mean degree of a node z=p(n-1) fixed (Poisson approximation of Binomial distribution)

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R

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- Given: property Q_{\downarrow} ("is connected", "has diameter xyz" etc.) of $G_{n,p}$: " $G_{n,p}$ has property Q with high probability": $P(Q|n,p) \rightarrow 1$ iff $n \rightarrow \infty$ (adaptated from [2] (which, in turn, is adaptated from [3]))
- In such models $G_{n,p}$ phase transitions exist for properties Q: "threshold function" q(n) (with $q(n) \rightarrow \infty$ if $n \rightarrow \infty$) so that:

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ਾ Kandom Graph Models: Poisson Graph

Example: giant component / connectedness of G_{n,p}

- Let u be the fraction of nodes that do not belong to giant component X == probability for a given node i to be not in X ⊾
- probability for a given node i (with assumed degree k) to be not in X
 == probability that none of its neighbors is in X
 == u^k
- \rightarrow u (k fixed) == u^k \rightarrow $u = \sum_{k=0}^{\infty} p_k u^k = \mathrm{e}^{-z} \sum_{k=0}^{\infty} \frac{(zu)^k}{k!} = \mathrm{e}^{z(u-1)}$
- \rightarrow fraction S of graph occupied by X is $S=1-u \rightarrow$

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Random Graph Models: Poisson Graph

Example: giant component / connectedness of G_{n,p}

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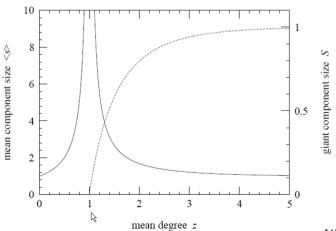
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Kandom Graph Models: Poisson Graph

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Kandom Graph Models: Poisson Graph

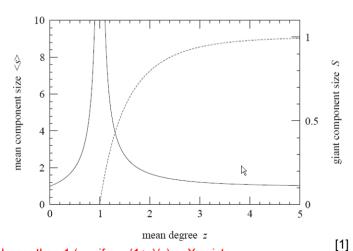
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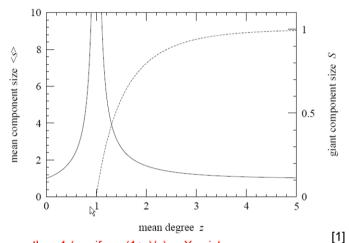
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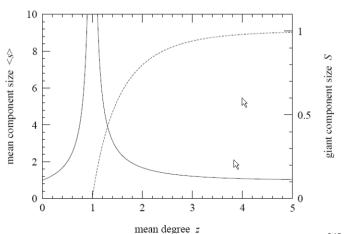
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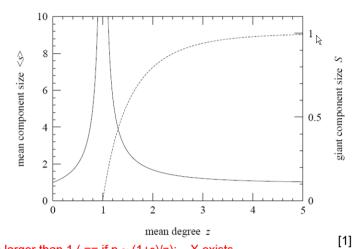
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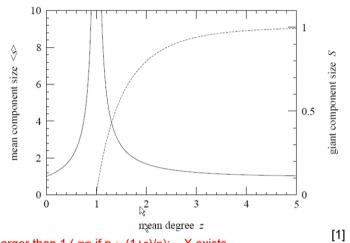


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