Script **generated by TTT**

Title: Seidl: Programmoptimierung (09.12.2013)

Date: Mon Dec 09 14:17:09 CET 2013

Duration: 86:50 min

Pages: 48

An expression e is called busy along a path π , if the expression e is evaluated before any of the variables $x \in Vars(e)$ is overwritten.

// backward analysis!

e is called very busy at u , if e is busy along every path $\pi : u \to^* stop$.

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// backward analysis!

e is called very busy at u , if e is busy along every path $\pi : u \to^* stop$.

Accordingly, we require:

$$\mathcal{B}[u] \;=\; \bigcap \{ [\![\pi]\!]^\sharp\, \emptyset \mid \pi : u \to^* stop \}$$

where for $\pi = k_1 \ldots k_m$:

$$[\![\pi]\!]^\sharp \;=\; [\![k_1]\!]^\sharp \circ \ldots \circ [\![k_m]\!]^\sharp$$

Our complete lattice is given by:

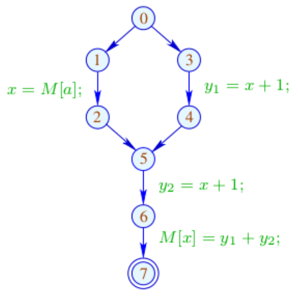
$$\mathbb{B} = 2^{Expr \setminus Vars} \qquad \text{with} \qquad \sqsubseteq \;=\; \supseteq$$

The effect $[\![k]\!]^\sharp$ of an edge $k = (u, lab, v)$ only depends on lab , i.e., $[\![k]\!]^\sharp = [\![lab]\!]^\sharp$ where:

$$\begin{array}{lll} [\![;]\!]^\sharp\, B & = & B \\ [\![Pos(e)]\!]^\sharp\, B & = & [\![Neg(e)]\!]^\sharp\, B \quad = \quad B \cup \{e\} \\ [\![x = e;]\!]^\sharp\, B & = & (B \setminus Expr_x) \cup \{e\} \\ [\![x = M[e];]\!]^\sharp\, B & = & (B \setminus Expr_x) \cup \{e\} \\ [\![M[e_1] = e_2;]\!]^\sharp\, B & = & B \cup \{e_1, e_2\} \end{array}$$

These effects are all distributive. Thus, the least solution of the constraint system yields precisely the MOP — given that *stop* is reachable from every program point :-)

Example:



7	$\emptyset$
6	$\{y_1 + y_2\}$
5	$\{x + 1\}$
4	$\{x + 1\}$
3	$\{x + 1\}$
2	$\{x + 1\}$
1	$\emptyset$
0	$\emptyset$

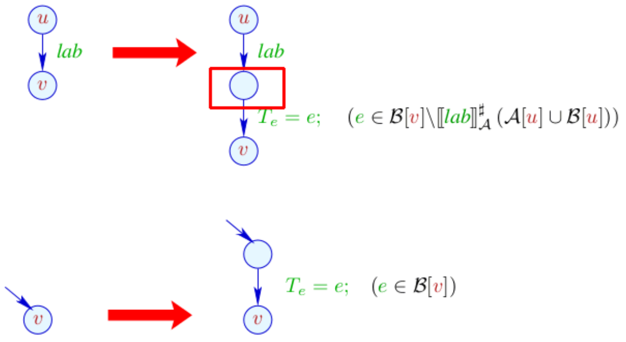
A point u is called safe for e, if $e \in \mathcal{A}[u] \cup \mathcal{B}[u]$, i.e., e is either available or very busy.

Idea:

- We insert computations of e such that e becomes available at all safe program points :-)
- We insert $T_e = e;$ after every edge (u, lab, v) with

$$e \in \mathcal{B}[v] \backslash [\![lab]\!]^\sharp_{\mathcal{A}}(\mathcal{A}[u] \cup \mathcal{B}[u])$$

Transformation 5.1:



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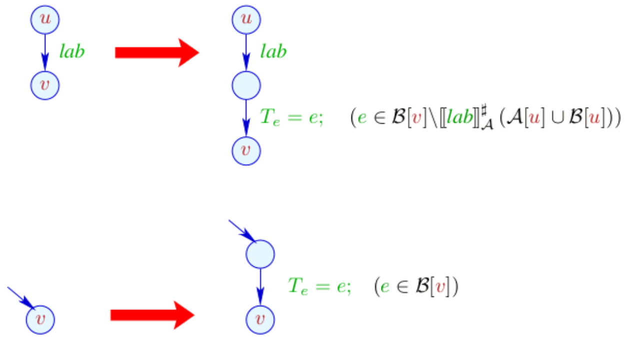
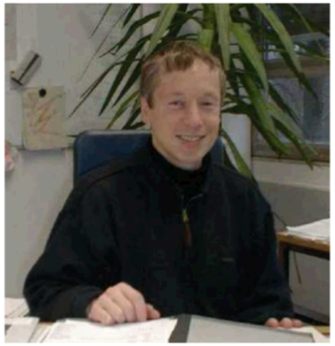


Transformation 5.2:

// analogously for the other uses of e

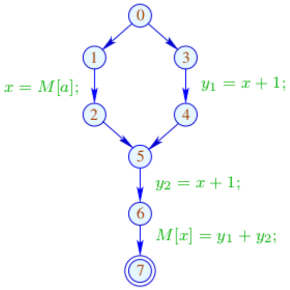
// at old edges of the program.



Bernhard Steffen, Dortmund

Jens Knoop, Wien

In the Example:



	$\mathcal{A}$	$\mathcal{B}$
0	$\emptyset$	$\emptyset$
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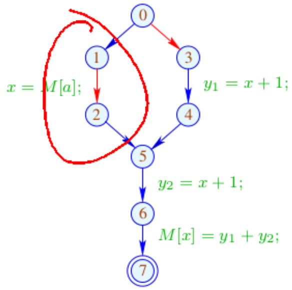

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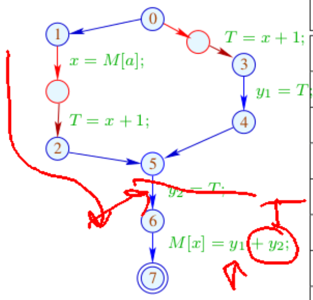
Im Example:



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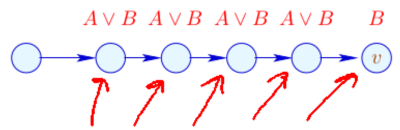
Correctness:

Let π denote a path reaching v after which a computation of an edge with e follows.

Then there is a maximal suffix of π such that for every edge $k = (u, lab, u')$ in the suffix:

$$e \in [\![lab]\!]^\sharp_{\mathcal{A}}(\mathcal{A}[u] \cup \mathcal{B}[u])$$



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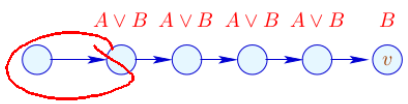

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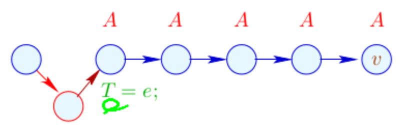
In particular, no variable in e receives a new value :-)

Then $T_e = e;$ is inserted before the suffix :-))



We conclude:

- Whenever the value of e is required, e is available :-)

  $\Longrightarrow$ correctness of the transformation

- Every $T = e;$ which is inserted into a path corresponds to an e which is replaced with T :-))

  $\Longrightarrow$ non-degradation of the efficiency

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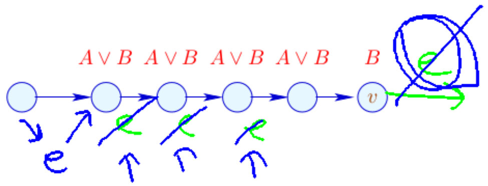

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1.8 Application: Loop-invariant Code

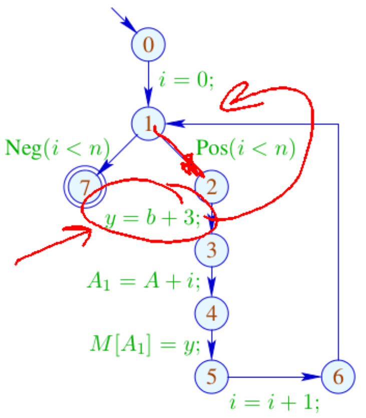
Example:

$$\text{for } (i = 0; i < n; i{+}{+})$$
$$a[i] = b + 3;$$

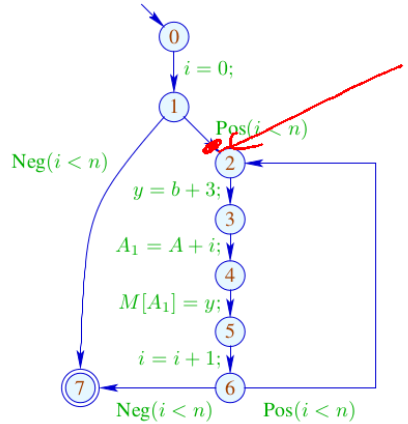
// The expression $b + 3$ is recomputed in every iteration :-(

// This should be avoided :-)

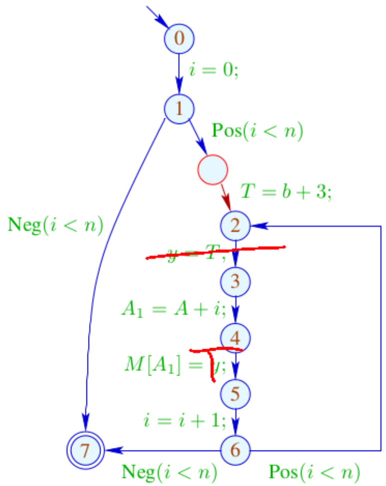
The Control-flow Graph:



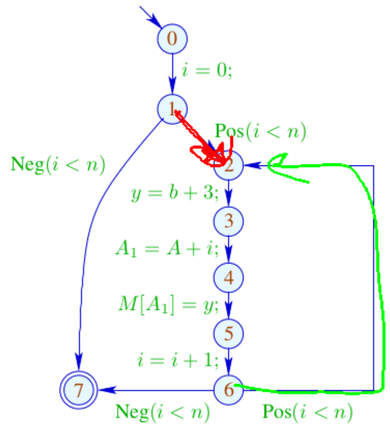
Idea: Transform into a do-while-loop ...



... now there is a place for $T = e;$:-)



Application of T5 (PRE) :



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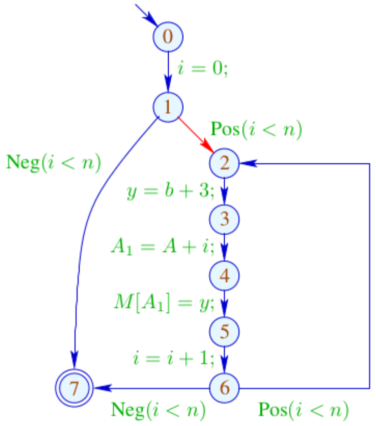

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Conclusion:

- Elimination of partial redundancies may move loop-invariant code out of the loop :-))
- This only works properly for do-while-loops :-(
- To optimize other loops, we transform them into do-while-loops before-hand:

while (b) $stmt$ $\Longrightarrow$ if (b)
do $stmt$
while (b);

$\Longrightarrow$ Loop Rotation

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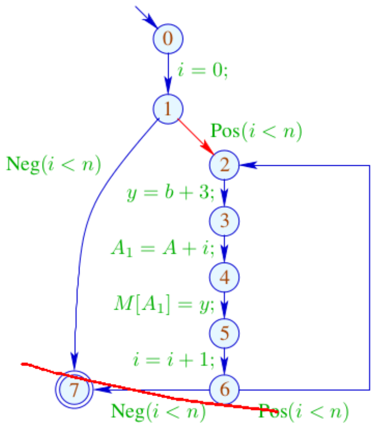

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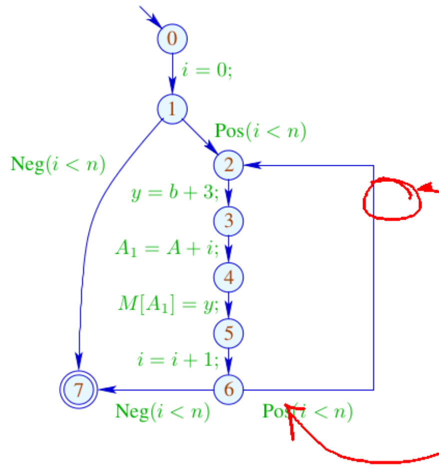

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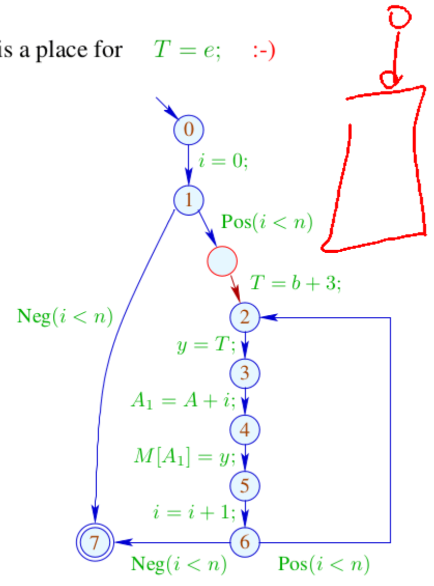

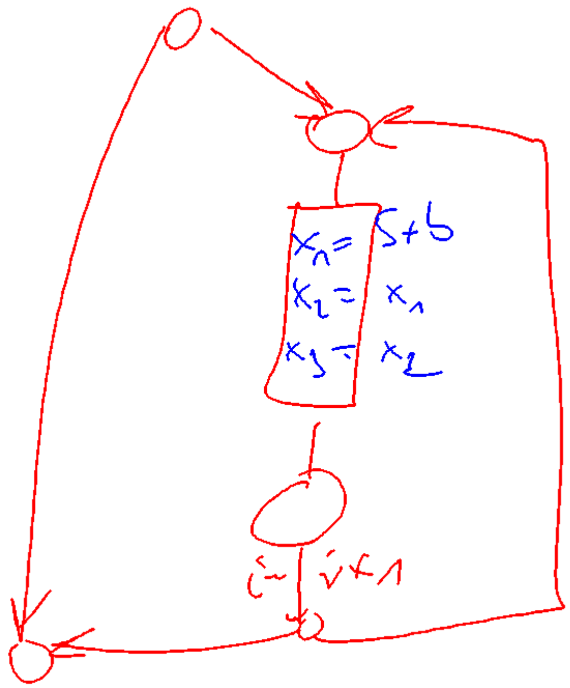

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```
while (b) stmt   ==>   if (b)
                           do stmt
                           while (b);
```

$\Longrightarrow$ Loop Rotation

Problem:

If we do not have the source program at hand, we must re-construct potential loop headers ;-)

$\Longrightarrow$ Pre-dominators

u pre-dominates v, if every path $\pi : start \to^* v$ contains u. We write: $u \Rightarrow v$.

"$\Rightarrow$" is reflexive, transitive and anti-symmetric :-)

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Computation:

We collect the nodes along paths by means of the analysis:

$$\mathbb{P} = 2^{Nodes} \quad , \qquad \sqsubseteq \; = \; \supseteq$$

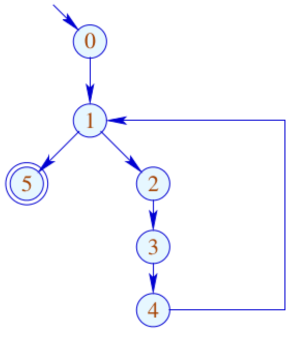
$$[\![(\_, \_, v)]\!]^\sharp \; P \quad = \boxed{P \cup \{v\}}$$

Then the set $\mathcal{P}[v]$ of pre-dominators is given by:

$$\mathcal{P}[v] = \bigcap \{ [\![\pi]\!]^\sharp \; \{start\} \mid \pi : start \to^* v \}$$

Since $[\![k]\!]^\sharp$ are distributive, the $\mathcal{P}[v]$ can computed by means of fixpoint iteration :-)

Example:



	$\mathcal{P}$
0	$\{0\}$
1	$\{0, 1\}$
2	$\{0, 1, 2\}$
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4	$\{0, 1, 2, 3, 4\}$
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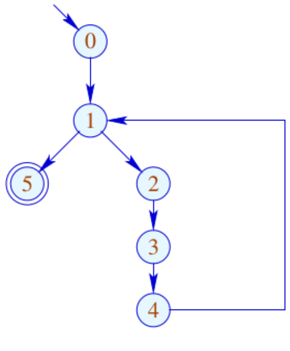

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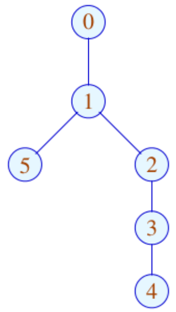
The partial ordering "⇒" in the example:



	$\mathcal{P}$
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3	$\{0, 1, 2, 3\}$
4	$\{0, 1, 2, 3, 4\}$
5	$\{0, 1, 5\}$

Apparently, the result is a tree :-)

In fact, we have:

Theorem:

Every node v has at most one immediate pre-dominator.

Proof:

Assume:

there are $u_1 \neq u_2$ which immediately pre-dominate v.

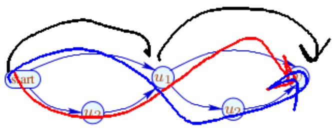
If $u_1 \Rightarrow u_2$ then u_1 not immediate.

Consequently, u_1, u_2 are incomparable :-)

Now for every $\pi : start \to^* v$:

$$\pi = \pi_1 \, \pi_2 \qquad \text{with} \qquad \begin{array}{l} \pi_1 : start \to^* u_1 \\ \pi_2 : u_1 \to^* v \end{array}$$

If, however, u_1, u_2 are incomparable, then there is path: $start \to^* v$ avoiding u_2 :



Observation:

The loop head of a while-loop pre-dominates every node in the body.

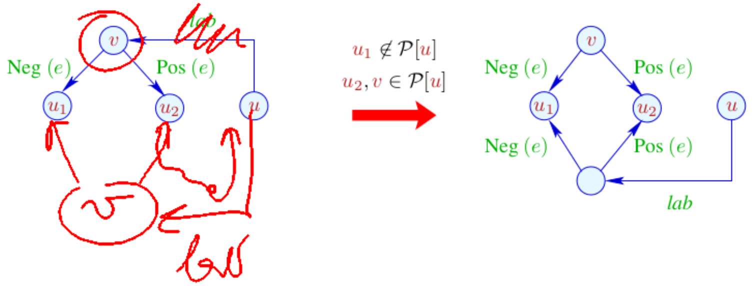
A back edge from the exit u to the loop head v can be identified through

$$v \in \mathcal{P}[u]$$

:-)

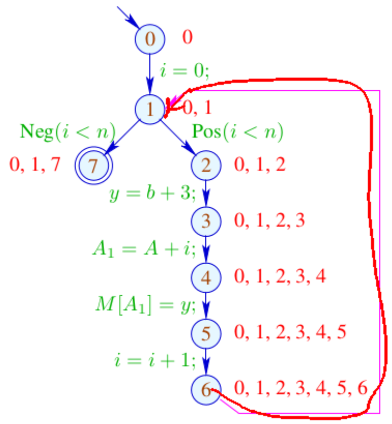
Accordingly, we define:

Transformation 6:



We duplicate the entry check to all back edges :-)

... in the Example:



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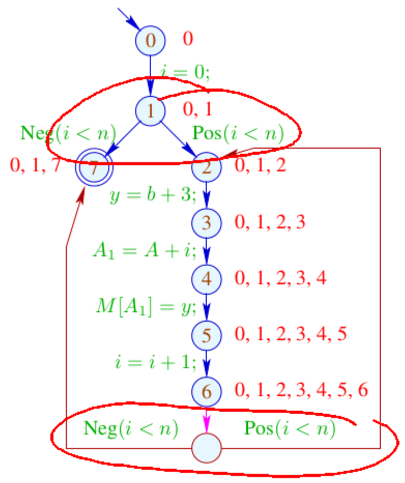
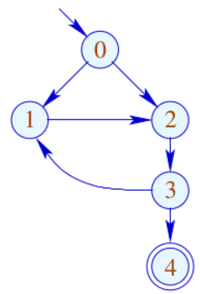
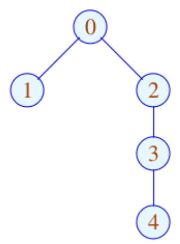


Warning:

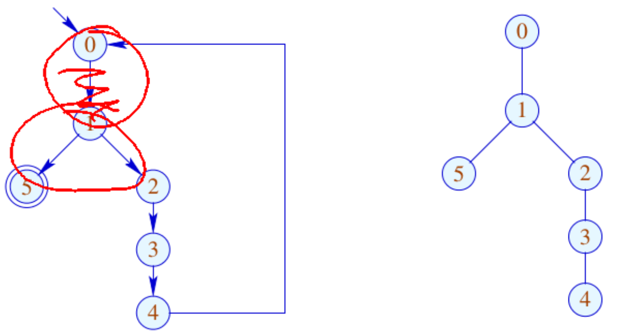
There are unusual loops which cannot be rotated:



Pre-dominators:



... but also common ones which cannot be rotated:



Here, the complete block between back edge and conditional jump should be duplicated :-(

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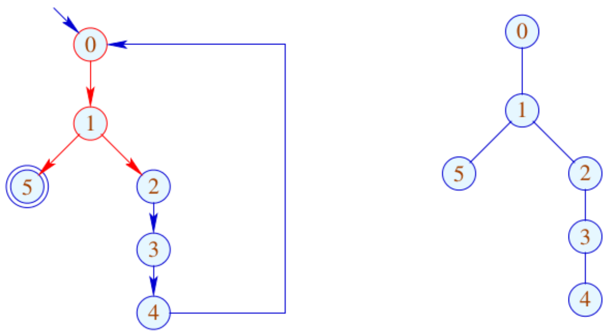

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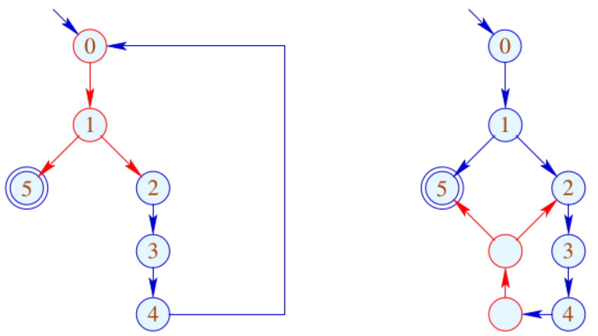

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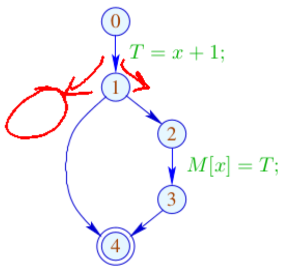
1.9 Eliminating Partially Dead Code

Example:



$x + 1$ need only be computed along one path ;-(

Idea: