Script generated by TTT

Title: Seidl: Programmoptimierung (04.02.2013)

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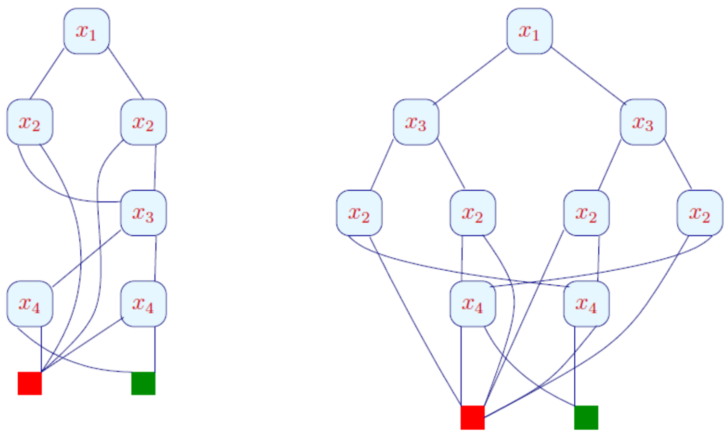
Discussion:

- Originally, BDDs have been developped for circuit verification.
- Today, they are also applied to the verification of software ...
- A system state is encoded by a sequence of bits.
- A BDD then describes the set of all reachable system states.
- Warning: Repeated application of Boolean operations may increase the size dramatically !
- The variable ordering may have a dramatic impact ...

Example: $(x_1 \leftrightarrow x_2) \wedge (x_3 \leftrightarrow x_4)$



Discussion (2):

- In general, consider the function:

$$(x_1 \leftrightarrow x_2) \wedge \ldots \wedge (x_{2n-1} \leftrightarrow x_{2n})$$

W.r.t. the variable ordering:

$$x_1 < x_2 < \ldots < x_{2n}$$

the BDD has $3n$ internal nodes.

W.r.t. the variable ordering:

$$x_1 < x_3 < \ldots < x_{2n-1} < x_2 < x_4 < \ldots < x_{2n}$$

the BDD has more than 2^n internal nodes !!

- A similar result holds for the implementation of Addition through BDDs.

addr mem

Discussion (3):

- Not all Boolean functions have small BDDs :-(
- Difficult functions:
  - multiplication;
  - indirect addressing ...

$\Longrightarrow$ data-intensive programs cannot be analyzed in this way :-(

Perspectives: Further Properties of Programs

Freeness: Is X_i possibly/always unbound ?

$\Longrightarrow$

If X_i is always unbound, no indexing for X_i is required :-)
If X_i is never unbound, indexing for X_i is complete :-)

Pair Sharing: Are X_i, X_j possibly bound to terms t_i, t_j with

$$Vars(t_i) \cap Vars(t_j) \neq \emptyset \quad ?$$

$\Longrightarrow$

Literals without sharing can be executed in parallel :-)

Remark:

Both analyses may profit from Groundness !

Example:

$$\begin{array}{lll}
\text{bigger}(X,Y) & \leftarrow & X = elephant, Y = horse \\
\text{bigger}(X,Y) & \leftarrow & X = horse, Y = donkey \\
\text{bigger}(X,Y) & \leftarrow & X = donkey, Y = dog \\
\text{bigger}(X,Y) & \leftarrow & X = donkey, Y = monkey \\
\text{is_bigger}(X,Y) & \leftarrow & \text{bigger}(X,Y) \\
\text{is_bigger}(X,Y) & \leftarrow & \text{bigger}(X,Z), \text{is_bigger}(Z,Y) \\
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\end{array}$$

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A more realistic Example:

$$\begin{array}{rcl} \mathsf{app}(X,Y,Z) & \leftarrow & X = [\,],\ Y = Z \\ \mathsf{app}(X,Y,Z) & \leftarrow & X = [H|X'],\ Z = [H|Z'],\ \mathsf{app}(X',Y,Z') \\ & \leftarrow & \mathsf{app}(X,[Y,c],[a,b,Z]) \end{array}$$

$X = a$
$Y = b$
$Z = c$

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Remark:

$[\,]$	$==$	the atom empty list
$[H\|Z]$	$==$	binary constructor application
$[a,b,Z]$	$==$	Abbreviation for: $[a\|[b\|[Z\|[\,]]]]$

Accordingly, a program p is constructed as follows:

$$\begin{array}{rcl} t & ::= & a \mid X \mid \_ \mid f(t_1,\ldots,t_n) \\ g & ::= & p(t_1,\ldots,t_k) \mid X = t \\ c & ::= & p(X_1,\ldots,X_k) \leftarrow \boxed{g_1,\ldots,g_r} \\ q & ::= & \leftarrow g_1,\ldots,g_r \\ p & ::= & \boxed{c_1 \ldots c_m q} \end{array}$$

- A term t either is an atom, a (possibly anonymous) variable or a constructor application.
- A goal g either is a literal, i.e., a predicate call, or a unification.
- A clause c consists of a head $p(X_1,\ldots,X_k)$ together with body consisting of a sequence of goals.
- A program consists of a sequence of clauses together with a sequence of goals as query.

Procedural View of PuP-Programs:

literal	$==$	procedure call
predicate	$==$	procedure
definition	$==$	body
term	$==$	value
unification	$==$	basic computation step
binding of variables	$==$	side effect

Warning: Predicate calls ...

- do not return results!
- modify the caller solely through side effects :-)
- may fail. Then, the following definition is tried $\Longrightarrow$ backtracking

Inefficiencies:

Backtracking: • The matching alternative must be searched for
$\Longrightarrow$ Indexing

- Since a successful call may still fail later, the stack can only be cleared if there are no pending alternatives.

Unification: • The translation possibly must switch between build and check several times.

- In case of unification with a variable, an Occur Check must be performed.

Type Checking: • Since Prolog is untyped, it must be checked at run-time whether or not a term is of the desired form.

- Otherwise, ugly errors could show up.

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Z = a , X = [Y | Z]

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Some Optimizations:

- Replacing last calls with jumps;
- Compile-time type inference;
- Identification of deterministic predicates ...

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$p(\text{out}, \text{in})$

Observation:

- In PuP, functions must be simulated through predicates.
- These then have designated input- and output parameters.
- Input parameters are those which are instantiated with a variable-free term whenever the predicate is called.

  These are also called ground.
- In the example, the first parameter of app is an input parameter.
- Unification with such a parameter can be implemented as pattern matching !
- Then we see that app in fact is deterministic !!!

5.1 Groundness Analysis

A variable X is called ground w.r.t. a program execution π starting program entry and entering a program point v, if X is bound to a variable-free term.

Goal:

- Find all variables which are ground whenever a particular program point is reached !
- Find all arguments of a predicate which are ground whenever the predicate is called !

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Idea:

- Describe groundness by values from $\mathbb{B}$:

  $1 \quad == \quad$ variable-free term;

  $0 \quad == \quad$ term which contains variables.

- A set of variable assignments is described by Boolean functions :-)

  $X \leftrightarrow Y \quad == \quad X$ is ground iff Y is ground.

  $X \wedge Y \quad == \quad X$ and Y are ground.

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Idea (cont.):

- The constant function 0 denotes an unreachable program point.
- Occurring sets of variable assignments are closed under substitution.

  This means that for every occurring function $\phi \neq 0$,

  $$\phi(1, \ldots, 1) = 1$$

  These functions are called positive.

- The set of all positive functions is called Pos.

  Ordering: $\phi_1 \sqsubseteq \phi_2 \quad$ if $\quad \phi_1 \Rightarrow \phi_2$.

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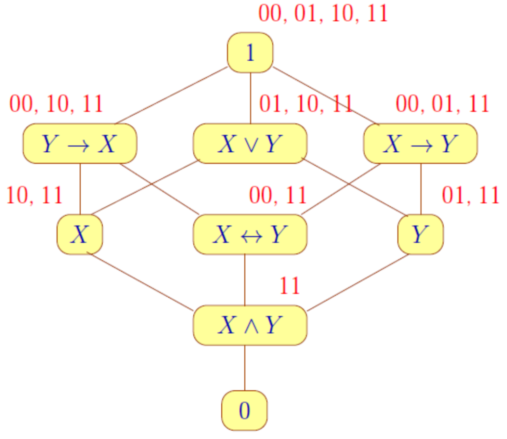

Remarks:

- Not all positive functions are monotonic !!!
- For k variables, there are $2^{2^k - 1} + 1$ many functions.
- The height of the complete lattice is 2^k.
- We construct an interprocedural analysis which for every predicate p determines a (monotonic) transformation

  $$[\![p]\!]^\sharp : \mathsf{Pos} \to \mathsf{Pos}$$

- For every clause, $p(X_1, \ldots, X_k) \Leftarrow g_1, \ldots, g_n$ we obtain the constraint:

  $$[\![p]\!]^\sharp \, \psi \quad \sqsupseteq \quad \exists X_{k+1}, \ldots, X_m . \, [\![g_n]\!]^\sharp \, (\ldots ([\![g_1]\!]^\sharp \, \psi) \ldots)$$

  // m number of clause variables

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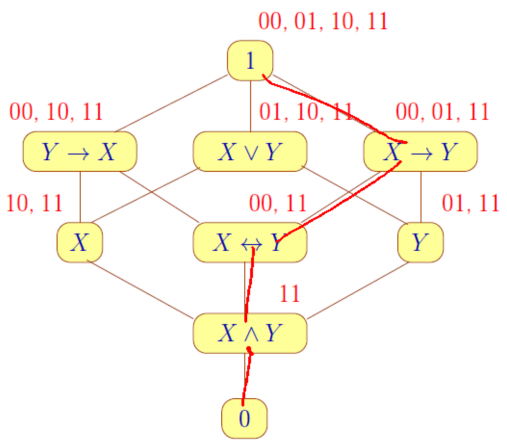

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Abstract Unification:

$$[\![X = t]\!]^\sharp \psi \quad = \quad \psi \wedge (X \leftrightarrow X_1 \wedge \ldots \wedge X_r)$$
$$\text{if} \quad Vars(t) = \{X_1, \ldots, X_r\}.$$

Abstract Literal:

$$[\![q(s_1, \ldots, s_k)]\!]^\sharp \psi \quad = \quad \mathsf{combine}^\sharp_{s_1,\ldots,s_k}(\psi, [\![q]\!]^\sharp (\cancel{\mathsf{enter}}^\sharp_{s_1,\ldots,s_k} \psi))$$

// analogous to $\boxed{\text{procedure}}$ call !!

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// analogous to procedure call !!

Thereby:

$$\mathsf{enter}^\sharp_{s_1,\ldots,s_k} \psi \quad = \quad \mathsf{ren}\, (\exists\, X_1, \ldots, X_m.\ [\![\bar{X}_1 = s_1, \ldots, \bar{X}_k = s_k]\!]^\sharp \psi)$$

$$\mathsf{combine}^\sharp_{s_1,\ldots,s_k}(\psi, \psi_1) \quad = \quad \exists\, \bar{X}_1, \ldots, \bar{X}_r.\ \psi \wedge [\![\bar{X}_1 = s_1, \ldots, \bar{X}_k = s_k]\!]^\sharp (\overline{\mathsf{ren}}\, \psi_1)$$

where

$$\exists\, X.\, \phi \quad = \quad \phi[0/X] \vee \phi[1/X]$$
$$\mathsf{ren}\, \phi \quad = \quad \phi[X_1/\bar{X}_1, \ldots, X_k/\bar{X}_k]$$

$$\overline{\mathsf{ren}}\, \phi \quad = \quad \phi[\bar{X}_1/X_1, \ldots, \bar{X}_r/X_r]$$