Script generated by TTT

Title: Seidl: Programmoptimierung (05.11.2012)

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Pages: 62

Summary and Application:

$(a \cup x) \setminus b$

→ The effects of edges of the analysis of availability of expressions are distributive:

$$\begin{aligned}(a \cup (x_1 \cap x_2)) \setminus b &= ((a \cup x_1) \cap (a \cup x_2)) \setminus b \\ &= ((a \cup x_1) \setminus b) \cap ((a \cup x_2) \setminus b)\end{aligned}$$

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→ If all effects of edges are distributive, then the MOP can be computed by means of the constraint system and RR-iteration. :-)

→ If not all effects of edges are distributive, then RR-iteration for the constraint system at least returns a safe upper bound to the MOP :-)

1.2 Removing Assignments to Dead Variables

Example:

$$\begin{array}{ll} 1: & \cancel{x = y + 2;} \\ 2: & y = 5; \\ 3: & x = y + 3; \end{array}$$

The value of x at program points $1, 2$ is over-written before it can be used.

Therefore, we call the variable x dead at these program points :-)

Note:

- $\rightarrow$ Assignments to dead variables can be removed ;-)
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Formal Definition:

The variable x is called live at u along the path π starting at u relative to a set X of variables either:

if $x \in X$ and π does not contain a definition of x; or:

if π can be decomposed into: $\pi = \pi_1\, k\, \pi_2$ such that:

- k is a use of x ; and
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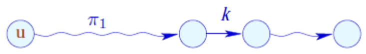
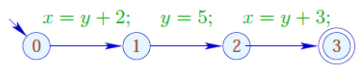


Thereby, the set of all defined or used variables at an edge $k = (\_, lab, \_)$ is defined by:

lab	used	defined
;	$\emptyset$	$\emptyset$
$\text{Pos}\,(e)$	$\mathit{Vars}\,(e)$	$\emptyset$
$\text{Neg}\,(e)$	$\mathit{Vars}\,(e)$	$\emptyset$
$x = e;$	$\mathit{Vars}\,(e)$	$\{x\}$
$x = M[e];$	$\mathit{Vars}\,(e)$	$\{x\}$
$M[e_1] = e_2;$	$\mathit{Vars}\,(e_1) \cup \mathit{Vars}\,(e_2)$	$\emptyset$

A variable x which is not live at u along π (relative to X) is called dead at u along π (relative to X).

Example:



where $X = \emptyset$. Then we observe:

	live	dead
0	$\{y\}$	$\{x\}$
1	$\emptyset$	$\{x, y\}$
2	$\{y\}$	$\{x\}$
3	$\emptyset$	$\{x, y\}$

The variable x is live at u (relative to X) if x is live at u along some path to the exit (relative to X). Otherwise, x is called dead at u (relative to X).

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Question:

How can the sets of all dead/live variables be computed for every u ???

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Idea:

For every edge $k = (u, \_, v)$, define a function $[\![k]\!]^\sharp$ which transforms the set of variables which are live at v into the set of variables which are live at u ...

Let $\mathbb{L} = 2^{Vars}$.

For $k = (\_, lab, \_)$, define $[\![k]\!]^\sharp = [\![lab]\!]^\sharp$ by:

$$\begin{array}{lll} [\![;]\!]^\sharp\, L & = & L \\ [\![\text{Pos}(e)]\!]^\sharp\, L & = & [\![\text{Neg}(e)]\!]^\sharp\, L \;=\; L \cup Vars(e) \\ [\![x = e;]\!]^\sharp\, L & = & (L \backslash \{x\}) \cup Vars(e) \\ [\![x = M[e];]\!]^\sharp\, L & = & (L \backslash \{x\}) \cup Vars(e) \\ [\![M[e_1] = e_2;]\!]^\sharp\, L & = & L \cup Vars(e_1) \cup Vars(e_2) \end{array}$$

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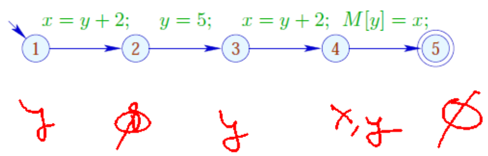
$[\![k]\!]^\sharp$ can again be composed to the effects of $[\![\pi]\!]^\sharp$ of paths $\pi = k_1 \ldots k_r$ by:

$$[\![\pi]\!]^\sharp = [\![k_1]\!]^\sharp \circ \ldots \circ [\![k_r]\!]^\sharp$$

We verify that these definitions are meaningful :-)



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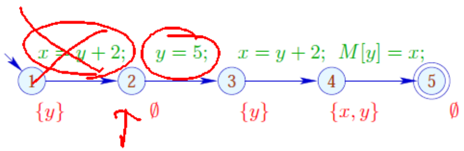

The set of variables which are live at u then is given by:

$$\mathcal{L}^*[u] \;=\; \bigcup \{ [\![\pi]\!]^\sharp \, X \mid \pi : u \to^* stop \}$$

$\cap$ $\exists$

... literally:

- The paths start in u :-)

  $\Longrightarrow$ As partial ordering for $\mathbb{L}$ we use $\sqsubseteq = \subseteq$.
- The set of variables which are live at program exit is given by the set X :-)

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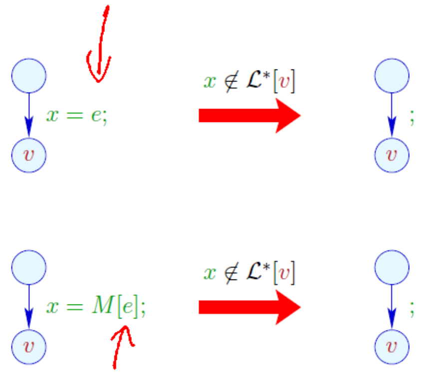

Correctness Proof:

→ Correctness of the effects of edges: If L is the set of variables which are live at the exit of the path π , then $[\![\pi]\!]^\sharp\, L$ is the set of variables which are live at the beginning of π :-)

→ Correctness of the transformation along a path: If the value of a variable is accessed, this variable is necessarily live. The value of dead variables thus is irrelevant :-)

→ Correctness of the transformation: In any execution of the transformed programs, the live variables always receive the same values :-))

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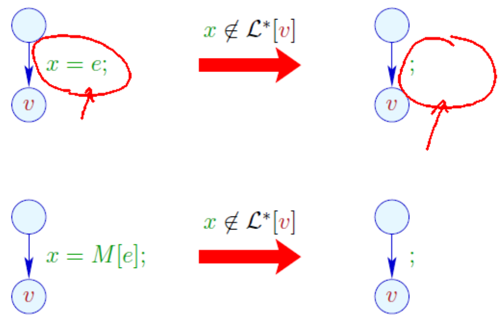

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Computation of the sets $\mathcal{L}^*[u]$:

(1) Collecting constraints:

$$\begin{array}{lll} \mathcal{L}[stop] & \supseteq & X \\ \mathcal{L}[u] & \supseteq & [\![k]\!]^\sharp\,(\mathcal{L}[v]) \qquad k = (u, \_, v) \quad \text{edge} \end{array}$$

(2) Solving the constraint system by means of RR iteration.
Since $\mathbb{L}$ is finite, the iteration will terminate :-)

(3) If the exit is (formally) reachable from every program point, then the smallest solution $\mathcal{L}$ of the constraint system equals $\mathcal{L}^*$ since all $[\![k]\!]^\sharp$ are distributive :-))

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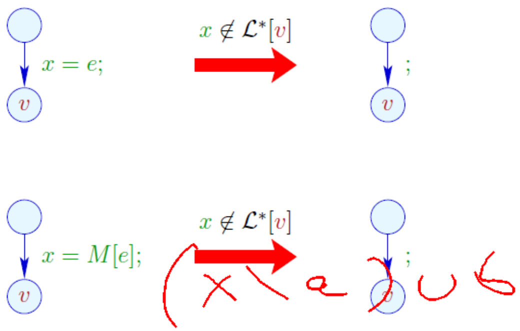

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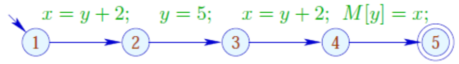
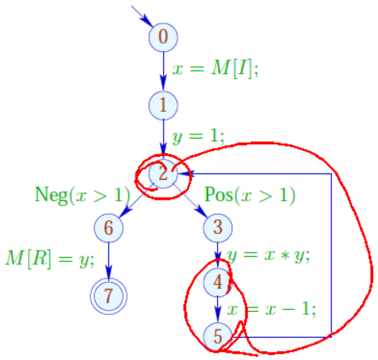


Example:



$$
\begin{array}{lcl}
\mathcal{L}[0] & \supseteq & (\mathcal{L}[1] \setminus \{x\}) \cup \{I\} \\
\mathcal{L}[1] & \supseteq & \mathcal{L}[2] \setminus \{y\} \\
\mathcal{L}[2] & \supseteq & (\mathcal{L}[6] \cup \{x\}) \cup (\mathcal{L}[3] \cup \{x\}) \\
\mathcal{L}[3] & \supseteq & (\mathcal{L}[4] \setminus \{y\}) \cup \{x, y\} \\
\mathcal{L}[4] & \supseteq & (\mathcal{L}[5] \cancel{\setminus \{x\}}) \cup \{x\} \\
\mathcal{L}[5] & \supseteq & \mathcal{L}[2] \\
\mathcal{L}[6] & \supseteq & \mathcal{L}[7] \cup \{y, R\} \\
\mathcal{L}[7] & \supseteq & \emptyset
\end{array}
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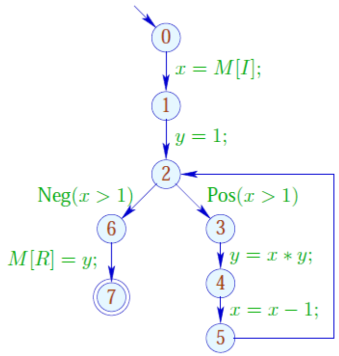

$\cup \{x\}$

	1	2
7	$\emptyset$	
6	$\{y, R\}$	
2	$\boxed{\{x, y, R\}}$	dito
5	$\{x, y, R\}$	
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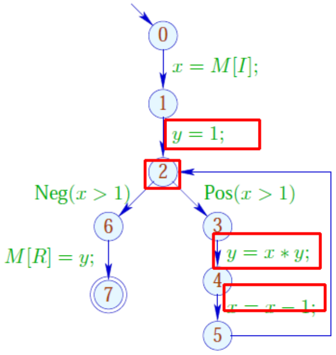

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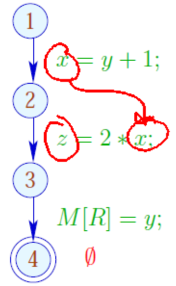
The left-hand side of no assignment is dead :-)

Caveat:

Removal of assignments to dead variables may kill further variables:



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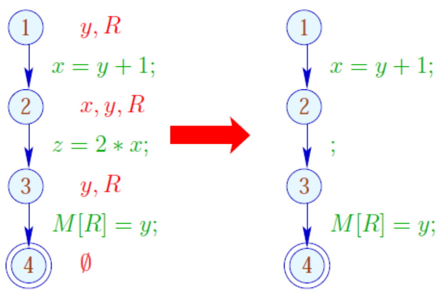

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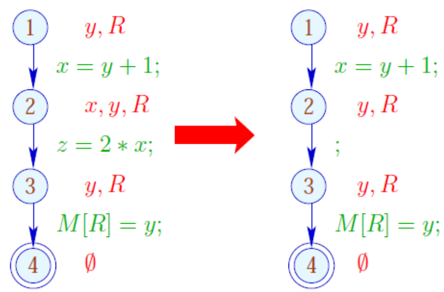

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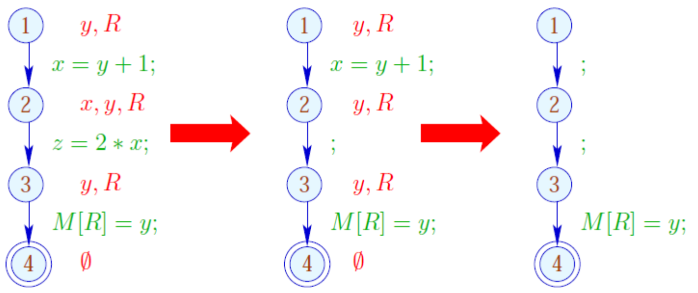

Re-analyzing the program is inconvenient :-(

Idea: Analyze true liveness!

x is called truely live at u along a path π (relative to X), either

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if π can be decomposed into $\pi = \pi_1\, k\, \pi_2$ such that:

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- π_1 does not contain any definition of x.

The set of truely used variables at an edge $k = (\_, lab, v)$ is defined as:

lab	truely used
;	$\emptyset$
Pos (e)	$Vars\,(e)$
Neg (e)	$Vars\,(e)$
$x = e;$	$Vars\,(e)$ $(*)$
$x = M[e];$	$Vars\,(e)$ $(*)$
$M[e_1] = e_2;$	$Vars(e_1) \cup Vars(e_2)$

$(*)$ – given that x is truely live ~~at v~~ :-)

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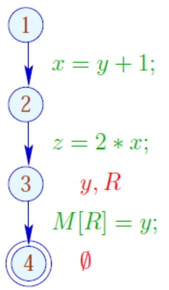

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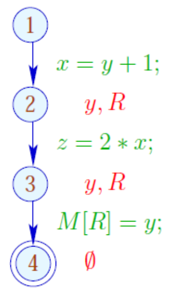

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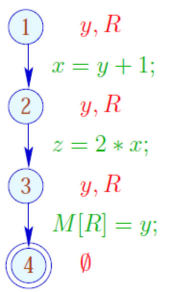

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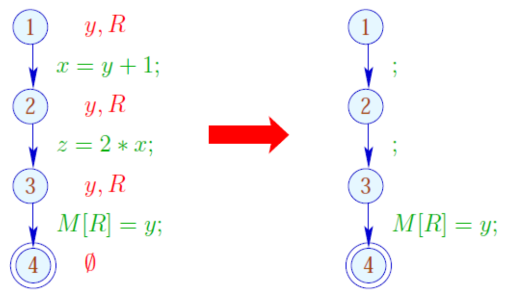

The Effects of Edges:

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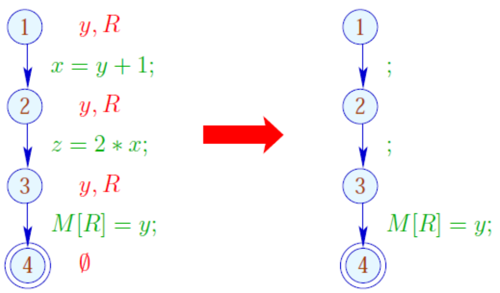

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- The effects of edges for truely live variables are more complicated than for live variables :-)
- Nonetheless, they are distributive !!

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- The effects of edges for truely live variables are more complicated than for live variables :-)
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To see this, consider for $\mathbb{D} = 2^U$, $f\, y = (u \in y)\, ?\, b : \emptyset$ We verify:

$$
\begin{array}{lcl}
f\,(y_1 \cup y_2) & = & (u \in y_1 \cup y_2)\, ?\, b : \emptyset \\
& = & (u \in y_1 \vee u \in y_2)\, ?\, b : \emptyset \\
& = & (u \in y_1)\, ?\, b : \emptyset \cup (u \in y_2)\, ?\, b : \emptyset \\
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$f_1(x) \cup f_2(x)$

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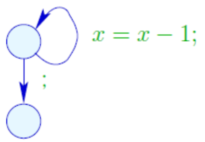
- True liveness detects more superfluous assignments than repeated liveness !!!



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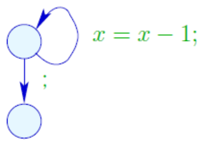
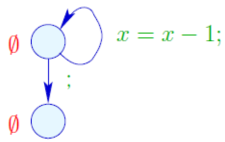


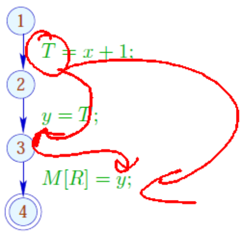
- True liveness detects more superfluous assignments than repeated liveness !!!

True Liveness:



1.3 Removing Superfluous Moves

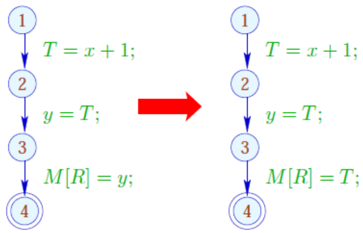
Example:



This variable-variable assignment is obviously useless :-(

1.3 Removing Superfluous Moves

Example:

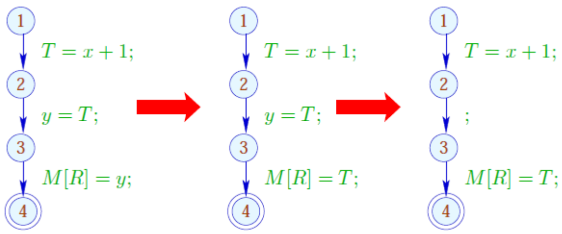


This variable-variable assignment is obviously useless :-(

Instead of y, we could also store T :-)

1.3 Removing Superfluous Moves

Example:



Advantage: Now, y has become dead :-))

Idea:

For each expression, we record the variable which currently contains its value :-)

We use: $\mathbb{V} = \mathit{Expr} \to 2^{\mathit{Vars}}$ ...

Idea:

For each expression, we record the variable which currently contains its value :-)

We use: $\mathbb{V} = \mathit{Expr} \to 2^{\mathit{Vars}}$ and define:

$$
\begin{array}{lllll}
[\![;]\!]^\sharp\, V & = & V & & \\
[\![\mathrm{Pos}(e)]\!]^\sharp\, V\, e' & = & [\![\mathrm{Neg}(e)]\!]^\sharp\, V\, e' & = & \begin{cases} \emptyset & \text{if } e' = e \\ V\, e' & \text{otherwise} \end{cases}
\end{array}
$$

...

// for Expr. Vals are diff. from Variable

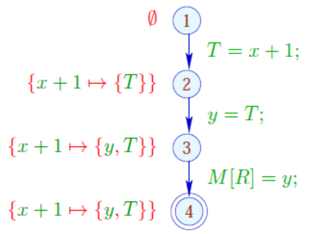
$$
\begin{array}{lll}
[\![x = c;]\!]^\sharp\, V\, e' & = & \begin{cases} (V\, c) \cup \{x\} & \text{if } e' = c \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases} \\
[\![x = y;]\!]^\sharp\, V\, e & = & \begin{cases} (V\, e) \cup \{x\} & \text{if } y \in V\, e \\ (V\, e) \backslash \{x\} & \text{otherwise} \end{cases} \\
[\![x = e;]\!]^\sharp\, V\, e' & = & \begin{cases} \{x\} & \text{if } e' = e \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases} \\
[\![x = M[c];]\!]^\sharp\, V\, e' & = & (V\, e') \backslash \{x\} \\
[\![x = M[y];]\!]^\sharp\, V\, e' & = & (V\, e') \backslash \{x\} \\
[\![x = M[e];]\!]^\sharp\, V\, e' & = & \begin{cases} \emptyset & \text{if } e' = e \\ (V\, e') \backslash \{x\} & \text{otherwise} \end{cases}
\end{array}
$$

// analogously for the diverse stores

$$
\begin{aligned}
[\![x = c;]\!]^\sharp\, V\, e' &= \begin{cases} (V\, c) \cup \{x\} & \text{if} \quad e' = c \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = y;]\!]^\sharp\, V\, e &= \begin{cases} (V\, e) \cup \{x\} & \text{if} \quad y \in V\, e \\ (V\, e)\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = e;]\!]^\sharp\, V\, e' &= \begin{cases} \{x\} & \text{if} \quad e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases} \\
[\![x = M[c];]\!]^\sharp\, V\, e' &= (V\, e')\backslash\{x\} \\
[\![x = M[y];]\!]^\sharp\, V\, e' &= (V\, e')\backslash\{x\} \\
[\![x = M[e];]\!]^\sharp\, V\, e' &= \begin{cases} \emptyset & \text{if } e' = e \\ (V\, e')\backslash\{x\} & \text{otherwise} \end{cases}
\end{aligned}
$$

// analogously for the diverse stores

In the Example:



$$
[\![M[e_1] = e_2;]\!]^\sharp\, V\, e = \begin{cases} \emptyset & \text{if } e \in \{e_1, e_2\} \\ V\, e' & \text{otherwise} \end{cases}
$$