

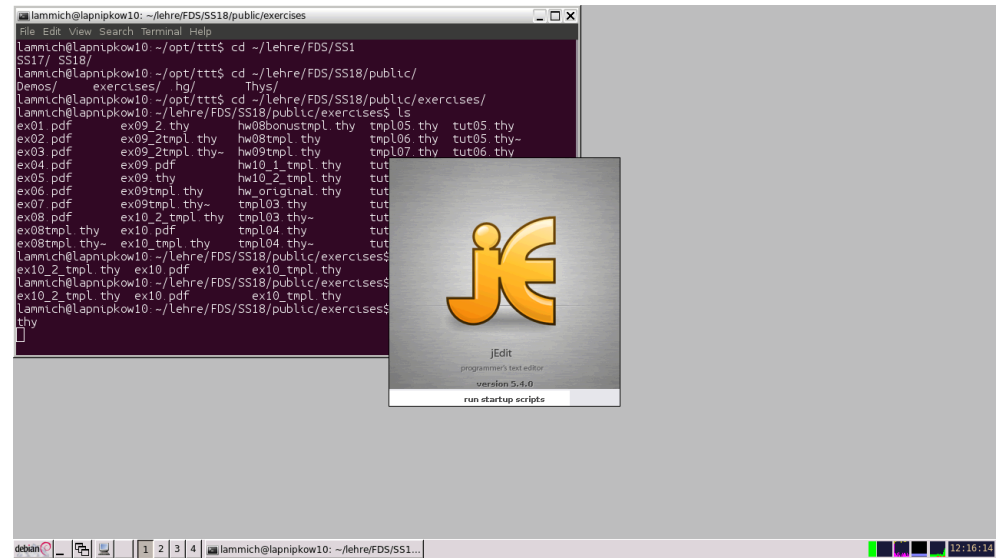
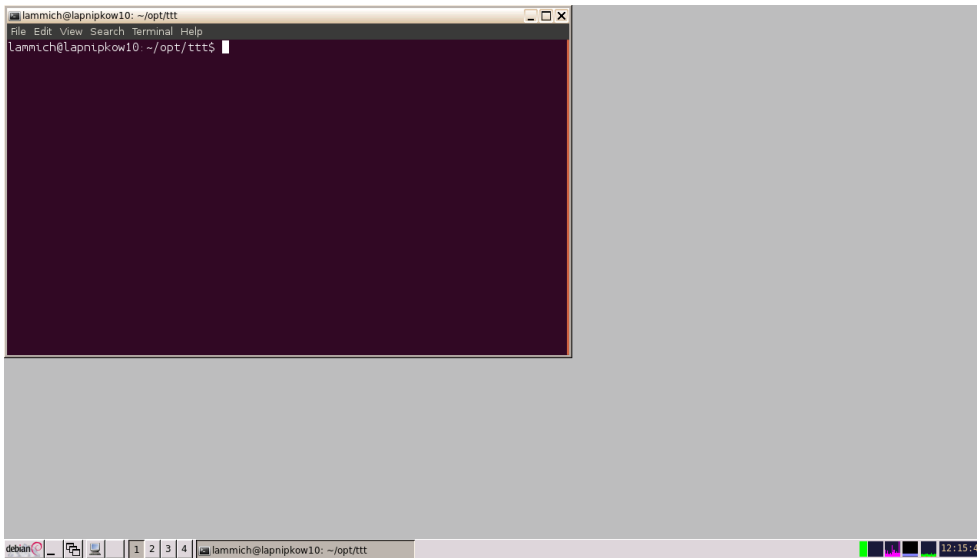
Script generated by TTT

Title: Lammich: FDS Tutorial (15.06.2018)

Date: Fri Jun 15 12:15:33 CEST 2018

Duration: 100:15 min

Pages: 90



```

1 (*<*)
2 theory ex10_tmpl
3 imports "HOL-Data_Structures.Tree23_Map"
4 begin
5 (*>*)
6 (*<*)
7   hide_const (open) Tree23.Leaf
8 (*>*)
9
10 text {* \ExerciseSheet{10}{15.-6.-2018} *}
11
12 text <\Exercise{Tries with 2-3-trees}
13
14 In this exercise, you shall develop a trie data structure for keys of
15 type @{typ "'a list"} (instead of @{typ "'bool list"}).
16
17 Thus, a node needs to store a map from '<a' to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}
20
21

```

4.1 (64/2422) Matches line 96: end (isabelle.isabelle,UTF-8-isabelle)Nmr o UG 510/1165MB 12:18 PM

```

1 (*<*)
2 theory ex10_tmpl
3 imports "HOL-Data_Structures.Tree23_Map"
4 begin
5 (*>*)
6 (*<*)
7   hide_const (open) Tree23.Leaf
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10 text {* \ExerciseSheet{10}{15.-6.-2018} *}
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12 text <\Exercise{Tries with 2-3-trees}
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14 In this exercise, you shall develop a trie data structure for keys of
15 type @{typ "'a list"} (instead of @{typ "'bool list"}).
16
17 Thus, a node needs to store a map from '<a' to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}
20
21

```

7.1 (82/2422) (isabelle.isabelle,UTF-8-isabelle)Nmr o UG 548/1165MB 12:23 PM

```

14 In this exercise, you shall develop a trie data structure for keys of
15 type @{typ "'a list"} (instead of @{typ "'bool list"}).
16
17 Thus, a node needs to store a map from '<a' to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}
20
21
22 datatype 'a trie = Leaf | Node bool "'a => 'a trie option"
23
24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin _ _ = undefined"
29
30 fun ins :: "'a list => 'a trie => 'a trie"
31 where
32   "ins _ _ = undefined"
33
34

```

22.1 (466/2429) (isabelle.isabelle,UTF-8-isabelle)Nmr o UG 53/1169MB 12:24 PM

```

59
60 text <The refined trie datatype>
61 datatype 'a trie = Leaf | Node bool ("<a x 'a trie'>") tree23"
62
63 text <Define an invariant for trie' and an abstraction function to trie.
64 Then define membership, insertion, and deletion, and show that they behave
65 correctly wrt. the abstract trie'. Finally, combine the correctness lemmas
66 to get a set interface based on 2-3-tree tries.
67
68 fun trie'_inv :: "'a::linorder trie' => bool" where
69   "trie'_inv _ = undefined"
70
71 fun trie'_alpha :: "'a::linorder trie' => 'a trie"
72 where
73   "trie'_alpha _ = undefined"
74
75 fun isin' :: "'a::linorder trie' => 'a list => bool"
76 where

```

68.25 (1858/2429) (isabelle.isabelle,UTF-8-isabelle)Nmr o UG 67/1169MB 12:26 PM

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
56 text <The refined trie datatype>
57 datatype 'a trie' = Leaf | Node bool ("('a x 'a trie)' tree23"
58
59 text <Define an invariant for trie' and an abstraction function to trie.
60 Then define membership, insertion, and deletion, and show that they behave
61 correctly wrt. the abstract trie'. Finally, combine the correctness lemmas
62 to get a set interface based on 2-3-tree tries.
63 >
64
65 fun trie'_inv :: "'a::linorder trie' => bool" where
66   "trie'_inv _ = undefined"
67
68 fun trie'_α :: "'a::linorder trie' => 'a trie"
69 where
70   "trie'_α _ = undefined"
71
72
73
74 fun isin' :: "'a::linorder trie' => 'a list => bool"
75 where

```

70.1 (1886/2429) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 674/1 69MB 12:27 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex10_tmpl.thy 12:27:04

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
57 datatype 'a trie' = Leaf | Node bool ("('a x 'a trie)' tree23"
58
59 text <Define an invariant for trie' and an abstraction function to trie.
60 Then define membership, insertion, and deletion, and show that they behave
61 correctly wrt. the abstract trie'. Finally, combine the correctness lemmas
62 to get a set interface based on 2-3-tree tries.
63 >
64
65 fun trie'_inv :: "'a::linorder trie' => bool" where
66   "trie'_inv _ = undefined"
67
68 fun trie'_α :: "'a::linorder trie' => 'a trie"
69 where
70   "trie'_α _ = undefined"
71
72
73
74 fun isin' :: "'a::linorder trie' => 'a list => bool"
75 where
76   "isin' _ _ = undefined"
77

```

59.1 (1474/2429) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 113/11 68MB 12:27 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex10_tmpl.thy 12:27:44

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
8 (*>*)
9
10 text {* \ExerciseSheet{10}{15.-6.-2018} *}
11
12 text <\Exercise{Tries with 2-3-trees}>
13
14 In this exercise, you shall develop a trie data structure for keys of
15 type @{typ "'a list"} (instead of @{typ "bool list"}).
16
17 Thus, a node needs to store a map from 'a' to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}'
20 >
21
22 datatype 'a trie = Leaf | Node bool "'a => 'a trie option"
23
24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin _ _ = undefined"

```

11.1 (164/2429) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 35/11 68MB 12:28 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex10_tmpl.thy 12:28:02

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
15 type @{typ "'a list"} (instead of @{typ "bool list"}).
16
17 Thus, a node needs to store a map from 'a' to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}'
20 >
21
22 datatype 'a trie = Leaf | Node bool "'a => 'a trie option"
23
24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin _ _ = undefined"
29
30 fun ins :: "'a list => 'a trie => 'a trie"
31 where
32   "ins _ _ = undefined"
33
34
35 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"

```

30.1 (702/2429) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 11/11 68MB 12:30 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - ex10_tmpl.thy 12:30:41

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
15 type @{typ "'a list"} (instead of @{typ "bool list"}).
16
17 Thus, a node needs to store a map from 'a to the next trie.
18
19 In a first step, we encode the map as @{typ '<a => 'b option>}
20
21
22 datatype 'a trie = Leaf | Node bool "'a => 'a trie option"
23
24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin _ _ = undefined"
29
30 fun ins :: "'a list => 'a trie => 'a trie"
31 where
32   "ins _ _ = undefined"
33
34
35 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
36
Output Query Sledgehammer Symbols
28.12 (685/2429) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 35% 1168MB 12:31 PM

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
21 datatype 'a trie = Leaf | Node bool "'a => 'a trie option"
22
23 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
24
25 fun isin :: "'a trie => 'a list => bool"
26 where
27   "isin Leaf _ = False"
28   | "isin (Node b m) [] ← b"
29   | "isin (Node b m) (k#ks) ← (case m k of None => False | Some t => isin t ks)"
30
31
32
33
34
35 fun ins :: "'a list => 'a trie => 'a trie"
36 where
37   "ins _ _ = undefined"
38
39
40 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
41 oops
42
Output Query Sledgehammer Symbols
34.3 (817/2545) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 52% 1259MB 12:35 PM

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
29 | "isin (Node b m) [] ← b"
30 | "isin (Node b m) (k#ks) ← (case m k of None => False | Some t => isin t ks)"
31
32
33 term "m(k←t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 fun ins :: "'a list => 'a trie => 'a trie"
38 where
39   "ins _ _ = undefined"
40
41
42 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
43
Proof state Auto update Update Search: 100%
"m(k ← t)"
:: "'a => 'b option"
Output Query Sledgehammer Symbols
39.9 (938/2606) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 55% 1310MB 12:38 PM

```

```

Isabelle2017 - Trie1.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Trie1.thy (~/lehre/FDS/SS18/public/Thys/)
25 [] => b |
26 k#ks => isin (if k then r else l) ks)"
27
28 fun insert :: "bool list => trie => trie" where
29 "insert [] Leaf = Node True (Leaf,Leaf)" |
30 "insert [] (Node b lr) = Node True lr" |
31 "insert (k#ks) Leaf =
32   Node False (if k then (Leaf, insert ks Leaf)
33                 else (insert ks Leaf, Leaf))" |
34 "insert (k#ks) (Node b (l,r)) =
35   Node b (if k then (l, insert ks r)
36             else (insert ks l, r))"
37
38 lemma ins_insert: "isin (insert as t) bs = (as = bs ∨ isin t bs)"
39
Proof state Auto update Update Search: 100%
consts
insert :: "bool list => trie => trie"
Found termination order: "(λp. size (snd p)) <+mlex*> (λp. size_list size (fst p)) <+mlex*> {}"
Output Query Sledgehammer Symbols
32.1 (564/4961) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 70% 110MB 12:39 PM

```

```

Isabelle2017 - Trie1.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
~/lehre/FDS/SS18/public/Thys/
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
Fun.thy ($ISABELLE_HOME/src/HOL)
Trie1.thy (~/lehre/FDS/SS18/public/Thys/)
27
28 fun insert :: "bool list => trie => trie" where
29 "insert [] Leaf = Node True (Leaf,Leaf)" |
30 "insert [] (Node b lr) = Node True lr" |
31 "insert (k#ks) Leaf =
32 Node False (if k then (Leaf, insert ks Leaf)
33 else (insert ks Leaf, Leaf))" |
34 "insert (k#ks) (Node b (l,r)) =
35 Node b (if k then (l, insert ks r)
36 else (insert ks l, r))"
37
38 Lemma isin_insert: "isin (insert as t) bs = (as = bs ∨ isin t bs)"

consts
insert :: "bool list => trie => trie"
Found termination order: "(λp. size (snd p) <+mlex*> (λp. size_list size (fst p) <+mlex*> {})"

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
~/lehre/FDS/SS18/public/exercises/
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
29 | "isin (Node b m) [] ← b"
30 | "isin (Node b m) (k#ks) ← (case m k of None => False | Some t => isin t ks)"
31
32
33 term "m(k←t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "λ_. None"
39
40 fun ins :: "'a list => 'a trie => 'a trie"
41 where
42 "ins _ _ = undefined"

"Map.empty"
:: "'a => 'b option"

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
~/lehre/FDS/SS18/public/exercises/
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33 term "m(k←t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "λ_. None"
39
40 fun ins :: "'a list => 'a trie => 'a trie"
41 where
42 "ins [] Leaf = undefined"
43
44
45 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
46 oops

consts
ins :: "'a list => 'a trie => 'a trie"
Missing patterns in function definition:
∧v va b. ins (v # va) b = undefined

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
~/lehre/FDS/SS18/public/exercises/
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33 term "m(k←t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "λ_. None"
39
40 fun ins :: "'a list => 'a trie => 'a trie"
41 where
42 "ins [] Leaf = undefined"
43
44
45 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
46 oops

consts
ins :: "'a list => 'a trie => 'a trie"
Missing patterns in function definition:
∧v va b. ins (v # va) b = undefined

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33 term "m(k→t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\. None"
39
40 fun ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
41 where
42   "ins [] Leaf = undefined"
43
44
45 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
46 oops

```

consts

```

ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
Missing patterns in function definition:
∧v va b. ins (v # va) b = undefined

```

42.20 (985/2646) (isabelle.isabelle.UTF-8-isabelle)NmroUG 254/1311MB 12:42 PM

```

Isabelle2017 - Trie1.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Trie1.thy (~/lehre/FDS/SS18/public/Thys/)
25 [] ⇒ b |
26 k#ks ⇒ isin (if k then r else l) ks"
27
28 fun insert :: "bool list ⇒ trie ⇒ trie" where
29 "insert [] Leaf = Node True (Leaf,Leaf)" |
30 "insert [] (Node b lr) = Node True lr" |
31 "insert (k#ks) Leaf =
32   Node False (if k then (Leaf, insert ks Leaf)
33     else (insert ks Leaf, Leaf))" |
34 "insert (k#ks) (Node b (l,r)) =
35   Node b (if k then (l, insert ks r)
36     else (insert ks l, r))"
37
38 lemma isin_insert: "isin (insert as t) bs = (as = bs ∨ isin t bs)"

```

consts

```

insert :: "bool list ⇒ trie ⇒ trie"
Found termination order: "(λp. size (snd p)) <+mlex*> (λp. size_list size (fst p)) <+mlex*> {}"

```

32.1 (564/4961) (isabelle.isabelle.UTF-8-isabelle)NmroUG 431/1311MB 12:43 PM

```

Isabelle2017 - Trie1.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Trie1.thy (~/lehre/FDS/SS18/public/Thys/)
27 fun insert :: "bool list ⇒ trie ⇒ trie" where
28 "insert [] Leaf = Node True (Leaf,Leaf)" |
29 "insert [] (Node b lr) = Node True lr" |
30 "insert (k#ks) Leaf =
31   Node False (if k then (Leaf, insert ks Leaf)
32     else (insert ks Leaf, Leaf))" |
33 "insert (k#ks) (Node b (l,r)) =
34   Node b (if k then (l, insert ks r)
35     else (insert ks l, r))"
36
37 lemma isin_insert: "isin (insert as t) bs = (as = bs ∨ isin t bs)"

```

consts

```

insert :: "bool list ⇒ trie ⇒ trie"
Found termination order: "(λp. size (snd p)) <+mlex*> (λp. size_list size (fst p)) <+mlex*> {}"

```

32.1 (564/4961) (isabelle.isabelle.UTF-8-isabelle)NmroUG 431/1311MB 12:43 PM

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33 term "m(k→t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\. None"
39
40 fun ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
41 where
42   "ins [] Leaf = Node True Map.empty"
43   | "ins [] (Node b m) = Node True b"
44
45 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"

```

Inner syntax error: unexpected end of input
Failed to parse prop

43.31 (1036/2690) (isabelle.isabelle.UTF-8-isabelle)NmroUG 658/1311MB 12:44 PM

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
33 term "m(k→t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\_. None"
39
40 fun ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
41 where
42   "ins [] Leaf = Node True Map.empty"
43 | "ins [] (Node b m) = Node True m"
44
45 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
46
consts
ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
Missing patterns in function definition:
∧v va b. ins (v # va) b = undefined

```

```

Isabelle2017 - Fun.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Fun.thy ($ISABELLE_HOME/src/HOL)
593 then show "bij_betw f A A'"
594 using * bij_betw_subset[of f "A U {b}" _ A] by blast
595 qed
596
597
598 subsection <Function Updating>
599
600 definition fun_upd :: "('a ⇒ 'b) ⇒ 'a ⇒ 'b ⇒ ('a ⇒ 'b)"
601 where "fun_upd f a b = (λx. if x = a then b else f x)"
602
603 nonterminal updbinds and updbind
604
605 syntax
606 "_updbind" :: "'a ⇒ 'a ⇒ updbind" ("(2_ :=/ _)")

```

```

Isabelle2017 - Fun.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Fun.thy ($ISABELLE_HOME/src/HOL)
593 then show "bij_betw f A A'"
594 using * bij_betw_subset[of f "A U {b}" _ A] by blast
595 qed
596
597 subsection <Function Updating>
598
599 definition fun_upd :: "('a ⇒ 'b) ⇒ 'a ⇒ 'b ⇒ ('a ⇒ 'b)"
600 where "fun_upd f a b = (λx. if x = a then b else f x)"
601
602 nonterminal updbinds and updbind
603
604 syntax
605 "_updbind" :: "'a ⇒ 'a ⇒ updbind" ("(2_ :=/ _)")

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\_. None"
39
40 fun ins :: "'a list ⇒ 'a trie ⇒ 'a trie"
41 where
42   "ins [] Leaf = Node True Map.empty"
43 | "ins [] (Node b m) = Node True m"
44 | "ins (k#ks) Leaf = {}"
45
46 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
47
Type unification failed: Clash of types "unit" and "_ trie"
Type error in application: incompatible operand type

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\_. None"
39
40 fun ins :: "'a list => 'a trie => 'a trie"
41 where
42   "ins [] Leaf = Node True Map.empty"
43 | "ins [] (Node b m) = Node True m"
44 | "ins (k#ks) Leaf = (let t = ins ks Leaf in )"
45
46
47 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"

```

Inner syntax error
Failed to parse prop

44.48 (1.091/2744) Input/output complete (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 505/1310MB 12:48 PM 12:48:14

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\_. None"
39
40 fun ins :: "'a list => 'a trie => 'a trie"
41 where
42   "ins [] Leaf = Node True Map.empty"
43 | "ins [] (Node b m) = Node True m"
44 | "ins (k#ks) Leaf = (let t = ins ks Leaf in Node False [k->t])"
45
46 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
47 oops

```

consts
ins :: "'a list => 'a trie => 'a trie"
Missing patterns in function definition:
 $\bigwedge v va vb vc. ins (v \# va) (Node vb vc) = undefined$

44.63 (1.106/2760) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 694/1310MB 12:48 PM 12:48:48

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
35 term "fun_upd m k (Some t)"
36
37 term Map.empty
38 term "\_. None"
39
40 term "[k->v]"
41 term "Map.empty (k->v)"
42
43
44
45 fun ins :: "'a list => 'a trie => 'a trie"
46 where
47   "ins [] Leaf = Node True Map.empty"
48 | "ins [] (Node b m) = Node True m"

```

"[k → v]"
:: "'a → 'b option"

40.15 (929/2802) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 691/1312MB 12:49 PM 12:49:45

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
41 term "Map.empty (k->v)"
42
43
44
45 fun ins :: "'a list => 'a trie => 'a trie"
46 where
47   "ins [] Leaf = Node True Map.empty"
48 | "ins [] (Node b m) = Node True m"
49 | "ins (k#ks) Leaf = Node False [k->ins ks Leaf]"
50 | "ins (k#ks) (Node b m) = {}"
51
52 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
53 oops
54

```

Inner syntax error: unexpected end of input
Failed to parse prop

50.32 (1.168/2819) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 697/1312MB 12:50 PM 12:50:57


```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
43
44
45 fun ins :: "'a list => 'a trie => 'a trie"
46 where
47   "ins [] Leaf = Node True Map.empty"
48 | "ins [] (Node b m) = Node True m"
49 | "ins (k#ks) Leaf = Node False [k->ins ks Leaf]"
50 | "ins (k#ks) (Node b m) = (
51   case m k of
52     None => Node b (m(k->ins ks Leaf))
53   | Some t => )"
54
55 lemma ins_correct: "isin (ins as t) bs = (as=bs v isin t bs)"
56
oops

```

Inner syntax error
Failed to parse prop

52.16 (1.201/2898) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 63/1314MB 12:52:11

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
48
49 | "ins [] (Node b m) = Node True m"
50 | "ins (k#ks) Leaf = Node False [k->ins ks Leaf]"
51 | "ins (k#ks) (Node b m) = (
52   case m k of
53     None => Node b (m(k->ins ks Leaf))
54   | Some t => Node b (m(k->ins ks t)))"
55
56
57 lemma ins_correct: "isin (ins as t) bs = (as=bs v isin t bs)"
58
59
60

```

proof (prove)
goal (1 subgoal):
1. ex10_tmpl.isin (ex10_tmpl.ins as t) bs = (as = bs v ex10_tmpl.isin t bs)

56.7 (1.283/2927) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 63/1315MB 12:54:32

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
48
49 | "ins [] (Node b m) = Node True m"
50 | "ins (k#ks) Leaf = Node False [k->ins ks Leaf]"
51 | "ins (k#ks) (Node b m) = (
52   case m k of
53     None => Node b (m(k->ins ks Leaf))
54   | Some t => Node b (m(k->ins ks t)))"
55
56 thm trie.induct...
57
58 lemma ins_correct: "isin (ins as t) bs = (as=bs v isin t bs)"
59
60

```

[?P Leaf; $\wedge x1 x2. (\wedge x2a x2aa. [x2a \in \text{range } x2; x2aa \in \text{set_option } x2a] \implies ?P x2aa) \implies ?P (\text{Node } x1 x2)] \implies ?P ?trie$

55.18 (1.287/2942) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 63/1315MB 12:55:PM

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
48
49 | "ins [] (Node b m) = Node True m"
50 | "ins (k#ks) Leaf = Node False [k->ins ks Leaf]"
51 | "ins (k#ks) (Node b m) = (
52   case m k of
53     None => Node b (m(k->ins ks Leaf))
54   | Some t => Node b (m(k->ins ks t)))"
55
56
57 thm trie.induct...
58
59 lemma ins_correct: "isin (ins as t) bs = (as=bs v isin t bs)"
60

```

[?P Leaf; $\wedge x1 x2. (\wedge x2a x2aa. [x2a \in \text{range } x2; x2aa \in \text{set_option } x2a] \implies ?P x2aa) \implies ?P (\text{Node } x1 x2)] \implies ?P ?trie$

56.7 (1.283/2956) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 63/1315MB 12:56:PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

51 case m k of
52   None => Node b (m(k->ins ks Leaf))
53   | Some t => Node b (m(k->ins ks t))"
54
55 term ran...
56 .....
57 thm trie.induct...
58 .....
59 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
60
61
62
63

```

"ran"
:: "('a => 'b option) => 'b set"

59.11 (1.331/2964) (isabelle.isabelle.UTF-8-isabelle)Nmro UG 796/1315MB 12:56 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

51 case m k of
52   None => Node b (m(k->ins ks Leaf))
53   | Some t => Node b (m(k->ins ks t))"
54
55 term ran...
56 .....
57 thm trie.induct...
58 .....
59 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
60
61
62
63

```

[?P Leaf; $\wedge x1 x2. (\wedge x2a x2aa. [x2a \in \text{range } x2; x2aa \in \text{set_option } x2a] \Rightarrow ?P x2aa) \Rightarrow ?P (\text{Node } x1 x2)]$
 $\Rightarrow ?P ?trie$

59.11 (1.302/2964) (isabelle.isabelle.UTF-8-isabelle)Nmro UG 830/1315MB 12:57 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

51 case m k of
52   None => Node b (m(k->ins ks Leaf))
53   | Some t => Node b (m(k->ins ks t))"
54
55 term ran...
56 .....
57 thm trie.induct...
58 thm ins.induct
59 .....
60 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
61
62
63

```

[?P [] Leaf; $\wedge b m. ?P [] (\text{Node } b m); \wedge k ks. ?P ks \text{ Leaf} \Rightarrow ?P (k \# ks) \text{ Leaf};$
 $\wedge k ks b m. [m k = \text{None} \Rightarrow ?P ks \text{ Leaf}; \wedge x2. m k = \text{Some } x2 \Rightarrow ?P ks x2] \Rightarrow ?P (k \# ks) (\text{Node } b m)]$
 $\Rightarrow ?P ?a0.0 ?a1.0$

58.17 (1.330/2981) Input/output complete (isabelle.isabelle.UTF-8-isabelle)Nmro UG 58/1317MB 12:57 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

51 case m k of
52   None => Node b (m(k->ins ks Leaf))
53   | Some t => Node b (m(k->ins ks t))"
54
55 term ran...
56 .....
57 thm trie.induct...
58 thm ins.induct
59 .....
60 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
61
62
63

```

[?P [] Leaf; $\wedge b m. ?P [] (\text{Node } b m); \wedge k ks. ?P ks \text{ Leaf} \Rightarrow ?P (k \# ks) \text{ Leaf};$
 $\wedge k ks b m. [m k = \text{None} \Rightarrow ?P ks \text{ Leaf}; \wedge x2. m k = \text{Some } x2 \Rightarrow ?P ks x2] \Rightarrow ?P (k \# ks) (\text{Node } b m)]$
 $\Rightarrow ?P ?a0.0 ?a1.0$

58.17 (1.330/2981) (isabelle.isabelle.UTF-8-isabelle)Nmro UG 75/1317MB 12:58 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin Leaf _ = False"
29 | "isin (Node b m) ks [] <-> b"
30 | "isin (Node b m) (k#ks) <-> (case m k of None => False | Some t => isin t ks)"
31
32
33 term "m(k->t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36

```

Type unification failed: Clash of types $_ \Rightarrow _$ and bool

Type error in application: operator not of function type

Operator: `isin (Node b m) ks :: bool`
 Operand: `[] :: 'a list`

29.25 (729/3057) Input/output complete (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 657/1 318MB 1:03 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin Leaf _ = False"
29 | "isin (Node b m) ks [] <-> b"
30 | "isin (Node b m) (k#ks) <-> (case m k of None => False | Some t => isin t ks)"
31
32
33 term "m(k->t)"
34 term "m(k:=Some t)"
35 term "fun_upd m k (Some t)"
36

```

Type unification failed: Clash of types $_ \Rightarrow _$ and bool

Type error in application: operator not of function type

Operator: `isin (Node b m) ks :: bool`
 Operand: `[] :: 'a list`

29.25 (729/3057) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 663/1 318MB 1:04 PM

Isabelle2017 - ex10_tmpl.thy

```

24 text <Define and prove correct membership, insertion and deletion (without shrinking the trie).>
25
26 fun isin :: "'a trie => 'a list => bool"
27 where
28   "isin Leaf _ = False"
29 | "isin (Node b m) ks <-> (case
30   ks of [] => b
31   | k#ks => (case m k of None => False | Some t => isin t ks))"
32
33
34 term "m(k->t)"
35 term "m(k:=Some t)"
36 term "fun_upd m k (Some t)"
37

```

consts

```

isin :: "'a trie => 'a list => bool"
Found termination order: "(λp. length (snd p)) <+mlex* {}"

```

31.12 (767/3059) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 620/1 320MB 1:05 PM

Isabelle2017 - ex10_tmpl.thy

```

62
63 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
64   apply (induction as t arbitrary: bs rule: ins.induct)
65   apply (auto split: list.split option.split)
66   done
67
68
69
70
71 fun delete :: "'a list => 'a trie => 'a trie" where

```

theorem ins_correct: `isin (ex10_tmpl.ins ?as ?t) ?bs = (?as = ?bs ∨ isin ?t ?bs)`

67.1 (1512/3090) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 5/1 319MB 1:07 PM

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
61
62
63 lemma ins_correct: "isin (ins as t) bs = (as=bs ∨ isin t bs)"
64 apply (induction as t arbitrary: bs rule: ins.induct)
65 apply (auto split: [])
66 done
67
68
69
70

proof (prove)
goal (4 subgoals):
1.  $\bigwedge bs. \text{case } bs \text{ of } [] \Rightarrow \text{True} \mid k \# ks' \Rightarrow \text{case None of None} \Rightarrow \text{False} \mid \text{Some } t \Rightarrow \text{isin } t \text{ } ks' \Rightarrow [] = bs$ 
2.  $\bigwedge b m bs. \text{isin } (\text{ex10_tmpl.ins } [] \text{ (Node } b \text{ m)}) \text{ } bs = ([] = bs \vee \text{isin } (\text{Node } b \text{ m}) \text{ } bs)$ 
3.  $\bigwedge k ks bs. (\bigwedge bs. \text{isin } (\text{ex10_tmpl.ins } ks \text{ Leaf}) \text{ } bs = (ks = bs \vee \text{isin Leaf } bs)) \Rightarrow \text{isin } (\text{ex10_tmpl.ins } (k \# ks) \text{ Leaf}) \text{ } bs = (k \# ks = bs \vee \text{isin Leaf } bs)$ 
4.  $\bigwedge k ks b m bs. (\bigwedge bs m k = \text{None} \Rightarrow \text{isin } (\text{ex10_tmpl.ins } ks \text{ Leaf}) \text{ } bs = (ks = bs \vee \text{isin Leaf } bs))$ 

Output Query Sledgehammer Symbols
65.24 (1.478/3070) Input/output complete (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 550.1319MB 1:07 PM

```

```

Isabelle2017 - Trie1.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Trie1.thy (~/lehre/FDS/SS18/public/Thys/)
44
45 fun delete :: "bool list  $\Rightarrow$  trie  $\Rightarrow$  trie" where
46 "delete ks Leaf = Leaf" |
47 "delete ks (Node b (l, r)) =
48 (case ks of
49 []  $\Rightarrow$  Node False (l, r) |
50 k#ks'  $\Rightarrow$  Node b (if k then (l, delete ks' r) else (delete ks' l, r)))"
51
52 Lemma "isin (delete as t) bs = (as  $\neq$  bs  $\wedge$  isin t bs)"
53 apply (induction as t arbitrary: bs rule: delete.induct)

consts
delete :: "bool list  $\Rightarrow$  trie  $\Rightarrow$  trie"
Found termination order: "( $\lambda p. \text{size } (\text{snd } p)$ ) <math>^* \text{mlex}</math> {})"

Output Query Sledgehammer Symbols
49.1 (1116/4961) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 08/1321MB 1:09 PM

```

```

Isabelle2017 - Fun.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Fun.thy ($ISABELLE_HOME/src/HOL/)
593
594 then show "bij_betw f A A'"
595 using * bij_betw_subset[of f "A U {b}" _ A] by blast
596 qed
597
598 subsection <Function Updating>
599
600 definition fun_upd []: "('a  $\Rightarrow$  'b)  $\Rightarrow$  'a  $\Rightarrow$  'b  $\Rightarrow$  ('a  $\Rightarrow$  'b)"
601 where "fun_upd f a b = ( $\lambda x. \text{if } x = a \text{ then } b \text{ else } f \text{ } x$ )"
602

Proof state Auto update Update Search: 100%
600.20 (17186/25593) null parsing complete, 0 error(s) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 35/1321MB 1:09 PM

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
71
72 fun delete :: "'a list  $\Rightarrow$  'a trie  $\Rightarrow$  'a trie" where
73 "delete _ Leaf = Leaf"
74 | "delete ks (Node b m) = (
75 case ks of
76 []  $\Rightarrow$  undefined
77 | k#ks  $\Rightarrow$  undefined)"
78
79 Lemma delete_correct: "isin (delete as t) bs = (as  $\neq$  bs  $\wedge$  isin t bs)"
80 oops

consts
delete :: "'a list  $\Rightarrow$  'a trie  $\Rightarrow$  'a trie"
Found termination order: "{}"

Output Query Sledgehammer Symbols
76.1 (1665/3184) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 67/1322MB 1:11 PM

```

```

Isabelle2017 - Fun.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
Fun.thy ($ISABELLE_HOME/src/HOL)
593 then show "bij_betw f A A'"
594 using * bij_betw_subset[of f "A U {b}" _ A] by blast
595 qed
596
597 subsection <Function Updating>
598
599 definition fun_upd :: "'a => 'b => 'a => 'b => ('a => 'b)"
600 where "fun_upd f a b = ( $\lambda$ x. if x = a then b else f x)"
601
602
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
600,20 (17186/25593) null parsing complete, 0 error(s) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 156/1322MB 1:11 PM

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
74 case ks of
75 [] => Node False m
76 | k#ks => (case m k of None => Node b m | Some t => Node b (m(k--delete ks t)))
77 )"
78
79 lemma delete_correct: "isin (delete as t) bs = (as ≠ bs ∧ isin t bs)"
80
81
82 text <Now refine the trie data structure to use 2-3-trees for the map.
83 Note: To make the provided interface more usable, we introduce
84
85
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
80,5 (1836/3247) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 159/1322MB 1:13 PM

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
74 case ks of
75 [] => Node False m
76 | k#ks => (case m k of None => Node b m | Some t => Node b (m(k--delete ks t)))
77 )"
78
79 lemma delete_correct: "isin (delete as t) bs = (as ≠ bs ∧ isin t bs)"
80 apply (induction as t arbitrary: bs rule: delete.induct)
81 apply (auto split: list.split option.split)
82 done
83
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
79,72 (1831/3360) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 160/1323MB 1:14 PM

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
73 | "delete ks (Node b m) = (
74 case ks of
75 [] => Node False m
76 | k#ks => (case m k of None => Node b m | Some t => Node b (m(k--delete ks t)))
77 )"
78
79 lemma delete_correct: "isin (delete as t) bs = (as ≠ bs ∧ isin t bs)"
80 apply (induction as t arbitrary: bs rule: delete.induct)
81 apply (auto split: list.split option.split)
82 done
83
Proof state Auto update Update Search: 100%
Output Query Sledgehammer Symbols
78,1 (1759/3360) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 160/1323MB 1:14 PM

```

```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
84 text <Now refine the trie data structure to use 2-3-trees for the map.
85 Note: To make the provided interface more usable, we introduce
86 some abbreviations here:
87 >
88
89 abbreviation "empty23 ≡ Tree23.Leaf"
90 abbreviation "inv23 t ≡ bal t ∧ sorted1 (inorder t)"
91
92 Lemmas map23_thms[simp] = map_empty map_update map_delete
93   invar_empty invar_update invar_delete

consts
inv23 :: "('a × 'b) tree23 ⇒ bool"

Output Query Sledgehammer Symbols
88.1 (2116/3360) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 168/1323MB 1:15 PM

```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
85 Note: To make the provided interface more usable, we introduce
86 some abbreviations here:
87 >
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89 Lemmas thms = map_empty map_update map_delete
90   invar_empty invar_update invar_delete
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```

```
Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
88 abbreviation "empty23 ≡ Tree23.Leaf"
89 abbreviation "inv23 t ≡ bal t ∧ sorted1 (inorder t)"
90
91
92 Lemmas map23_thms[simp] = map_empty map_update map_delete
93   invar_empty invar_update invar_delete
94
95 text <The refined trie datatype>

theorem map23_thms:
  lookup empty23 = Map.empty
  inv23 ?m ⇒ lookup (update ?a ?b ?m) = lookup ?m(?a ↦ ?b)
  inv23 ?m ⇒ lookup (Tree23_Map.delete ?a ?m) = (lookup ?m)(?a := None)
  inv23 empty23
  inv23 ?m ⇒ inv23 (update ?a ?b ?m)
  inv23 ?m ⇒ inv23 (Tree23_Map.delete ?a ?m)
```

```
Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
93   invar_empty invar_update invar_delete
94
95 text <The refined trie datatype>
96 datatype 'a trie' = Leaf' | Node' bool "(('a × 'a) trie)" tree23"
97
98 text <Define an invariant for 'trie' and an abstraction function to trie.
99 Then define membership, insertion, and deletion, and show that they behave
100 correctly wrt. the abstract 'trie'. Finally, combine the correctness lemmas
```

```
Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
94 text <The refined trie datatype>
95 datatype 'a trie' = Leaf' | Node' bool "(('a × 'a) trie)" tree23"
96
97
98 text <Define an invariant for 'trie' and an abstraction function to trie.
99 Then define membership, insertion, and deletion, and show that they behave
100 correctly wrt. the abstract 'trie'. Finally, combine the correctness lemmas
101 to get a set interface based on 2-3-tree tries.
```

```
Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
95 text <The refined trie datatype>
96 datatype 'a trie' = Leaf' | Node' bool "(('a × 'a) trie)" tree23"
97
98 text <Define an invariant for 'trie' and an abstraction function to trie.
99 Then define membership, insertion, and deletion, and show that they behave
100 correctly wrt. the abstract 'trie'. Finally, combine the correctness lemmas
101 to get a set interface based on 2-3-tree tries.
```


Isabelle2017 - ex10_tmpl.thy (modified)

```

102 >
103
104 fun trie'_inv :: "'a::linorder trie' ⇒ bool" where
105   "trie'_inv Leaf' = True"
106 | "trie'_inv (Node' b m) ↔ ".
107
108 fun trie'_α :: "'a::linorder trie' ⇒ 'a trie"
109 where

```

Inner syntax error: unexpected end of input
Failed to parse prop

106.19 (2781/3388) (isabelle,isabelle,UTF-8-Isabelle)Nmro UG 99/1 324MB 1:26 PM 13:26:54

Isabelle2017 - ex10_tmpl.thy (modified)

```

102 >
103
104 fun trie'_inv :: "'a::linorder trie' ⇒ bool" where
105   "trie'_inv Leaf' = True"
106 | "trie'_inv (Node' b m) ↔ ".
107
108 fun trie'_α :: "'a::linorder trie' ⇒ 'a trie"
109 where

```

Inner syntax error: unexpected end of input
Failed to parse prop

106.28 (2790/3389) Input/output complete (isabelle,isabelle,UTF-8-Isabelle)Nmro UG L22/1 324MB 1:27 PM 13:27:13

Isabelle2017 - ex10_tmpl.thy (modified)

```

102 >
103
104 fun trie'_inv :: "'a::linorder trie' ⇒ bool" where
105   "trie'_inv Leaf' = True"
106 | "trie'_inv (Node' b m) ↔ inv".
107
108 fun trie'_α :: "'a::linorder trie' ⇒ 'a trie"
109 where

```

Inner syntax error: unexpected end of input
Failed to parse prop

106.31 (2793/3392) (isabelle,isabelle,UTF-8-Isabelle)Nmro UG :25/1 324MB 1:27 PM 13:27:26

Isabelle2017 - ex10_tmpl.thy

```

102 >
103
104 fun trie'_inv :: "'a::linorder trie' ⇒ bool" where
105   "trie'_inv Leaf' = True"
106 | "trie'_inv (Node' b m) ↔ inv23 m ∧ (∀t<ran (lookup m). trie'_inv t)"
107
108 fun trie'_α :: "'a::linorder trie' ⇒ 'a trie"
109 where

```

consts
trie'_inv :: "'a trie' ⇒ bool"
Unfinished subgoals:
(a, 1, <):
1. $\bigwedge m x. x \in \text{ran (lookup } m) \Rightarrow \text{size } x < \text{Suc (size_tree23 } (\lambda x. \text{Suc (size (snd } x))) m)$
(a, 1, <=):
1. $\bigwedge m x. x \in \text{ran (lookup } m) \Rightarrow \text{size } x \leq \text{Suc (size_tree23 } (\lambda x. \text{Suc (size (snd } x))) m)$
Calls:
a) Node' b m -> x
Measures:

106.69 (2831/3431) (isabelle,isabelle,UTF-8-Isabelle)Nmro UG :25/1 324MB 1:29 PM 13:29:00

Isabelle2017 - ex10_tmpl.thy

```

102 >
103
104 fun trie_inv :: "'a::linorder trie' => bool" where
105   "trie_inv Leaf = True"
106 | "trie_inv (Node' b m) <-> inv23 m ^ (∀t<ran (lookup m). trie_inv t)"
107
108 fun trie'_α :: "'a::linorder trie' => 'a trie"
109 where

```

consts
 trie_inv :: "'a trie' => bool"
 Unfinished subgoals:
 (a, 1, <):
 1. $\bigwedge m x. x \in \text{ran}(\text{lookup } m) \implies \text{size } x < \text{Suc}(\text{size_tree23 } (\lambda x. \text{Suc}(\text{size}(\text{snd } x))) m)$
 (a, 1, <=):
 1. $\bigwedge m x. x \in \text{ran}(\text{lookup } m) \implies \text{size } x \leq \text{Suc}(\text{size_tree23 } (\lambda x. \text{Suc}(\text{size}(\text{snd } x))) m)$
 Calls:
 a) Node' b m -> x
 Measures:
 1. size

106.69 (2831/3431) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 476/1324MB 1:31 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

103
104
105 Lemma [simp]: "x ∈ ran (lookup m) => size x < Suc (size_tree23 (λx. Suc (size (snd x))) m)"
106   apply (induction m arbitrary: x) apply (auto simp: ran_def split: if_splits)
107 --
108
109 fun trie_inv :: "'a::linorder trie' => bool" where
110   "trie_inv Leaf = True"

```

proof (prove)
 goal (5 subgoals):
 1. $\bigwedge m1 a b m2 x aa.$
 $\bigwedge \lambda x. \exists a. \text{lookup } m1 a = \text{Some } x \implies \text{size } x < \text{Suc}(\text{size_tree23 } (\lambda x. \text{Suc}(\text{size}(\text{snd } x))) m1);$
 $\bigwedge \lambda x. \exists a. \text{lookup } m2 a = \text{Some } x \implies \text{size } x < \text{Suc}(\text{size_tree23 } (\lambda x. \text{Suc}(\text{size}(\text{snd } x))) m2); aa < a;$
 $\text{lookup } m1 aa = \text{Some } x \implies \text{size } x < \text{Suc}(\text{Suc}(\text{Suc}(\text{size } b + \text{size_tree23 } (\lambda xa. \text{Suc}(\text{size}(\text{snd } xa))) m1 + \text{size_tree23 } (\lambda xa. \text{Suc}(\text{size}(\text{snd } xa))) m2)))$
 2. $\bigwedge m1 a b m2 x aa.$
 $\bigwedge \lambda x. \exists a. \text{lookup } m1 a = \text{Some } x \implies \text{size } x < \text{Suc}(\text{size_tree23 } (\lambda x. \text{Suc}(\text{size}(\text{snd } x))) m1);$

107.3 (2858/3605) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 526/1325MB 1:32 PM

Isabelle2017 - ex10_tmpl.thy

```

105 Lemma [simp]: "x ∈ ran (lookup m) => size x < Suc (size_tree23 (λx. Suc (size (snd x))) m)"
106   apply (induction m arbitrary: x) apply (auto simp: ran_def split: if_splits)
107   apply force+ done
108
109 fun trie_inv :: "'a::linorder trie' => bool" where
110   "trie_inv Leaf = True"
111 | "trie_inv (Node' b m) <-> inv23 m ^ (∀t<ran (lookup m). trie_inv t)"
112

```

consts
 trie_inv :: "'a trie' => bool"
 Found termination order: "size < *mlex* {}"

108.1 (2876/3622) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 587/1325MB 1:33 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

107   apply force+ done
108
109 fun trie_inv :: "'a::linorder trie' => bool" where
110   "trie_inv Leaf = True"
111 | "trie_inv (Node' b m) <-> inv23 m ^ (∀t<ran (lookup m). trie_inv t)"
112
113 fun trie'_α :: "'a::linorder trie' => 'a trie"
114 where
115   "trie'_α Leaf = Lea"

```

consts
 trie'_α :: "'a trie' => 'a trie"
 Missing patterns in function definition:
 $\bigwedge v va. \text{trie}'_\alpha (\text{Node } v va) = \text{undefined}$
 Found termination order: "{}"

115.23 (3101/3620) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 46/1325MB 1:33 PM

Isabelle2017 - ex10_tmpl.thy (modified)

```

108 fun trie'_inv :: "'a::linorder trie' => bool" where
109   "trie'_inv Leaf' = True"
110 | "trie'_inv (Node' b m) <-> inv23 m ^ (Vteran (lookup m). trie'_inv t)"
111
112
113 fun trie'_α :: "'a::linorder trie' => 'a trie"
114 where
115   "trie'_α Leaf' = Leaf"
116 | "trie'_α (Node' b t) = Node b []".
117
118
119
120 fun isin' :: "'a::linorder trie' => 'a list => bool"

```

116.33 (3136/3659) (isabelle.isabelle.UTF-8-Isabelle)tmr o UG 31.01.1325MB 1:35 PM

Output Query Sledgehammer Symbols

Type unification failed: Clash of types " $_ \Rightarrow _$ " and " $_ \text{ trie}$ "

Type error in application: incompatible operand type

Operator: $op = (trie'_\alpha (Node' b t)) :: 'a \text{ trie} \Rightarrow bool$

Operand: $Node' b :: (??'a \Rightarrow ??'a \text{ trie option}) \Rightarrow ??'a \text{ trie}$

Isabelle2017 - ex10_tmpl.thy (modified)

```

108 fun trie'_inv :: "'a::linorder trie' => bool" where
109   "trie'_inv Leaf' = True"
110 | "trie'_inv (Node' b m) <-> inv23 m ^ (Vteran (lookup m). trie'_inv t)"
111
112
113 fun trie'_α :: "'a::linorder trie' => 'a trie"
114 where
115   "trie'_α Leaf' = Leaf"
116 | "trie'_α (Node' b t) = Node b []".
117
118
119
120 fun isin' :: "'a::linorder trie' => 'a list => bool"

```

116.35 (3138/3661) (isabelle.isabelle.UTF-8-Isabelle)tmr o UG 31.01.1325MB 1:35 PM

Output Query Sledgehammer Symbols

Type unification failed: Clash of types "unit" and " $_ \Rightarrow _$ "

Type error in application: incompatible operand type

Operator: $Node' b :: (??'a \Rightarrow ??'a \text{ trie option}) \Rightarrow ??'a \text{ trie}$

Operand: $() :: unit$

Isabelle2017 - ex10_tmpl.thy

```

108 fun trie'_inv :: "'a::linorder trie' => bool" where
109   "trie'_inv Leaf' = True"
110 | "trie'_inv (Node' b m) <-> inv23 m ^ (Vteran (lookup m). trie'_inv t)"
111
112
113 fun trie'_α :: "'a::linorder trie' => 'a trie"
114 where
115   "trie'_α Leaf' = Leaf"
116 | "trie'_α (Node' b t) = Node b (map_option trie'_α o lookup t)".
117
118
119
120 fun isin' :: "'a::linorder trie' => 'a list => bool"

```

116.53 (3136/3690) (isabelle.isabelle.UTF-8-Isabelle)tmr o UG 31.01.1325MB 1:36 PM

Output Query Sledgehammer Symbols

consts

$trie'_\alpha :: 'a \text{ trie} \Rightarrow 'a \text{ trie}$

Unfinished subgoals:

(a, 1, <):

1. $\bigwedge t a x. \text{Some } a = \text{lookup } t x \Rightarrow \text{size } a < \text{Suc } (\text{size_tree23 } (\lambda x. \text{Suc } (\text{size } (\text{snd } x)))) t$

(a, 1, <=):

Isabelle2017 - ex10_tmpl.thy

```

124 "isin' Leaf' _ = False"
125 | "isin' (Node' b m) ks <-> (case
126   ks of [] => b
127   | k#ks' => (case lookup m k of None => False | Some t => isin' t ks'))"
128
129
130 fun ins' :: "'a::linorder list => 'a trie => 'a trie"
131 where
132   "ins' _ _ = undefined"
133
134
135 fun delete' :: "'a::linorder list => 'a trie => 'a trie"

```

132.1 (3638/4007) (isabelle.isabelle.UTF-8-Isabelle)tmr o UG 31.01.1325MB 1:38 PM

Output Query Sledgehammer Symbols

consts

$isin' :: 'a \text{ trie} \Rightarrow 'a \text{ list} \Rightarrow bool$

Found termination order: " $(\lambda p. \text{length } (\text{snd } p)) <^* mlex^* \{ \}$ "


```

Isabelle2017 - ex10_tmpl.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
146 assumes "trie'_inv t"
147 shows "trie'_α (ins' xs t) = ins xs (trie'_α t)"
148 "trie'_inv (ins' xs t)"
149 using assms apply (induction xs t rule: ins'.induct )
150 apply (auto split: option.splits)
151
152
153
154 Lemma ins'_correct: "trie'_inv t
155
proof (prove)
goal (2 subgoals):
1.  $\forall k \text{ ks m x2.}$ 

$$\begin{aligned} & [\forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2a) = \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2a); \\ & \forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\text{inv}} (\text{ins}' \text{ ks } x2a); \text{bal m}; \text{sorted1 (inorder m)}; \\ & \forall x \in \text{ran (lookup m)}. \text{trie}'_{\text{inv}} x; \text{lookup m k} = \text{Some } x2] \\ & \Rightarrow (\text{map\_option } \text{trie}'_{\alpha} \circ \text{lookup m})(k \mapsto \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2)) = (\text{map\_option } \text{trie}'_{\alpha} \circ \text{lookup m})(k \mapsto \\ & \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2)) \end{aligned}$$

2.  $\forall k \text{ ks m x2 x.}$ 

$$\begin{aligned} & [\forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2a) = \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2a); \\ & \forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\text{inv}} (\text{ins}' \text{ ks } x2a); \text{bal m}; \text{sorted1 (inorder m)}; \\ & \forall x \in \text{ran (lookup m)}. \text{trie}'_{\text{inv}} x; \text{lookup m k} = \text{Some } x2; x \in \text{ran (lookup m)(k \mapsto \text{ins}' \text{ ks } x2)}] \end{aligned}$$


```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
147 shows "trie'_α (ins' xs t) = ins xs (trie'_α t)"
148 "trie'_inv (ins' xs t)"
149 using assms apply (induction xs t rule: ins'.induct )
150 apply (auto split: option.splits intro!: ext)
151 --
152
153
154 Lemma ins'_correct: "trie'_inv t
155
proof (prove)
goal (2 subgoals):
1.  $\forall k \text{ ks m x2.}$ 

$$\begin{aligned} & [\forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2a) = \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2a); \\ & \forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\text{inv}} (\text{ins}' \text{ ks } x2a); \text{bal m}; \text{sorted1 (inorder m)}; \\ & \forall x \in \text{ran (lookup m)}. \text{trie}'_{\text{inv}} x; \text{lookup m k} = \text{Some } x2] \\ & \Rightarrow \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2) = \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2) \end{aligned}$$

2.  $\forall k \text{ ks m x2 x.}$ 

$$\begin{aligned} & [\forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\alpha} (\text{ins}' \text{ ks } x2a) = \text{ex10\_tmpl.ins ks (trie}'_{\alpha} x2a); \\ & \forall x2a. [x2 = x2a; \text{trie}'_{\text{inv}} x2a] \Rightarrow \text{trie}'_{\text{inv}} (\text{ins}' \text{ ks } x2a); \text{bal m}; \text{sorted1 (inorder m)}; \\ & \forall x \in \text{ran (lookup m)}. \text{trie}'_{\text{inv}} x; \text{lookup m k} = \text{Some } x2; x \in \text{ran (lookup m)(k \mapsto \text{ins}' \text{ ks } x2)}] \end{aligned}$$


```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
148 "trie'_inv (ins' xs t)"
149 using assms apply (induction xs t rule: ins'.induct )
150 apply (auto split: option.splits simp: ran_def intro!: ext)
151 thm ext
152 done
153
154
155 Lemma ins'_correct: "trie'_inv t
156

$$(\forall x. ?f x = ?g x) \Rightarrow ?f = ?g$$


```

```

Isabelle2017 - ex10_tmpl.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex10_tmpl.thy (~/lehre/FDS/SS18/public/exercises/)
148 "trie'_inv (ins' xs t)"
149 using assms apply (induction xs t rule: ins'.induct )
150 apply (auto split: option.splits simp: ran_def intro!: ext)
151 thm ext
152 done
153
154
155 Lemma ins'_correct: "trie'_inv t
156

$$(\forall x. ?f x = ?g x) \Rightarrow ?f = ?g$$


```

lammich@lapnikow10: ~/lehre/FDS/SS18/public/exercises

lammich@lapnikow10: ~/opt/ttt\$ cd ~/Lehre/FDS/SS1

ex10.pdf

93.95%

Technische Universität München SS 2018
 Institut für Informatik 15. 6. 2018
 Prof. Tobias Nipkow, Ph.D.
 Dr. Peter Lammich

Functional Data Structures
 Exercise Sheet 10

Exercise 10.1 Tries with 2-3-trees

In this exercise, you shall develop a trie data structure for keys of type *a list* (instead of *bool list*).

Thus, a node needs to store a map from *a* to the next trie.

In a first step, we encode the map as *a* \Rightarrow *b option*

datatype *a trie* = *Leaf* | *Node* *bool* "*a* \Rightarrow *b trie*"

Define and prove correct membership, insertion and deletion (without shrinking the trie).

fun *isin* :: "*a trie* \Rightarrow *a list* \Rightarrow *bool*"

fun *ins* :: "*a list* \Rightarrow *a trie* \Rightarrow *a trie*"

lemma *is_correct*: "*isin* (*ins* *as* *t*) *bs* = (*as* *as* *v* *isin* *t* *bs*)"

fun *delete* :: "*a list* \Rightarrow *a trie* \Rightarrow *a trie*" **where**

lemma *delete_correct*: "*isin* (*delete* *as* *t*) *bs* = (*as* \neq *bs* *isin* *t* *bs*)"

Now refine the trie data structure to use 2-3-trees for the map. Note: To make the provided interface more usable, we introduce some abbreviations here:

abbreviation *empty23* = *Tree23* *Leaf*

5 thy
 5 thy-
 6 thy
 6 thy-
 7 thy

h@lapnikow10: ~/lehre/FDS/SS18/public

View Search Terminal Help

@lapnikow10: ~/opt/ttt\$ cd ~/lehre/FDS/
 / SS17/ SS18/ web/
 @lapnikow10: ~/opt/ttt\$ cd ~/lehre/FDS/SS18/public/
 @lapnikow10: ~/lehre/FDS/SS18/public\$ evince exercises/ex10.pdf

ex10.pdf

2 of 4

203.54%

It is meant to store keys of the same length only. Thus, the *Node* constructor stores inner nodes, and there are two types of leaves, *LeafF* if this path is not in the set, and *LeafT* if it is in the set.

Define an invariant *is_trie* *N t* that states that all keys in *t* have length *N*, and that there are no superfluous nodes, i.e., no nodes of the form *Node (LeafF, LeafF)*.

fun *is_trie* :: "*nat* \Rightarrow *trie* \Rightarrow *bool*"

Hint: The following should evaluate to true!

value "*is_trie* 42 *LeafF*"

value "*is_trie* 2 (*Node* (*LeafF*, *Node* (*LeafT*, *LeafF*)))"

Whereas these should be false

value "*is_trie* 42 *LeafT*" — Wrong key length

value "*is_trie* 2 (*Node* (*LeafT*, *Node* (*LeafT*, *LeafF*)))" — Wrong key length

value "*is_trie* 1 (*Node* (*LeafT*, *Node* (*LeafF*, *LeafF*)))" — Superfluous node

Define membership, insert, and delete functions, and prove them correct!

fun *isin* :: "*trie* \Rightarrow *bool list* \Rightarrow *bool*"

fun *ins* :: "*bool list* \Rightarrow *trie* \Rightarrow *trie*"

1 2 3 4 lammich@lapnikow10: ~/le... Isabelle2017 - ex10_tmpl.thy lammich@lapnikow10: ~/le... ex10.pdf 13:48:08

ex10.pdf

2 of 4

203.54%

value "*is_trie* 42 *LeafT*" — Wrong key length

value "*is_trie* 2 (*Node* (*LeafT*, *Node* (*LeafT*, *LeafF*)))" — Wrong key length

value "*is_trie* 1 (*Node* (*LeafT*, *Node* (*LeafF*, *LeafF*)))" — Superfluous node

Define membership, insert, and delete functions, and prove them correct!

fun *isin* :: "*trie* \Rightarrow *bool list* \Rightarrow *bool*"

fun *ins* :: "*bool list* \Rightarrow *trie* \Rightarrow *trie*"

2

1 2 3 4 lammich@lapnikow10: ~/le... Isabelle2017 - ex10_tmpl.thy lammich@lapnikow10: ~/le... ex10.pdf 13:50:53

ex10.pdf

3 of 4

203.54%

lemma *enum_correct*:
 "*set* (*enum* *t*) = { *xs*. *isin* *t* *xs* }" **and** "*sorted_wrt* *op* < (*enum* *t*)"

Note that Booleans are ordered by *False* < *True*, and that we imported *List.lexord*, which defines a lexicographic ordering on lists, if the elements are ordered.

value "[*True*, *True*, *False*] < [*True*, *True*, *True*, *True*]"

Homework 10.3 Be Original!

Submission until Friday, 13. 7. 2017, 11:59am.

Develop a nice Isabelle formalization yourself!

- This homework goes in parallel to other homeworks for the rest of the lecture period. From next sheet on, we will reduce regular homework load a bit, such that you have a time-frame of 3 weeks with reduced regular homework load.

1 2 3 4 lammich@lapnikow10: ~/le... Isabelle2017 - ex10_tmpl.thy lammich@lapnikow10: ~/le... ex10.pdf 13:52:04

