

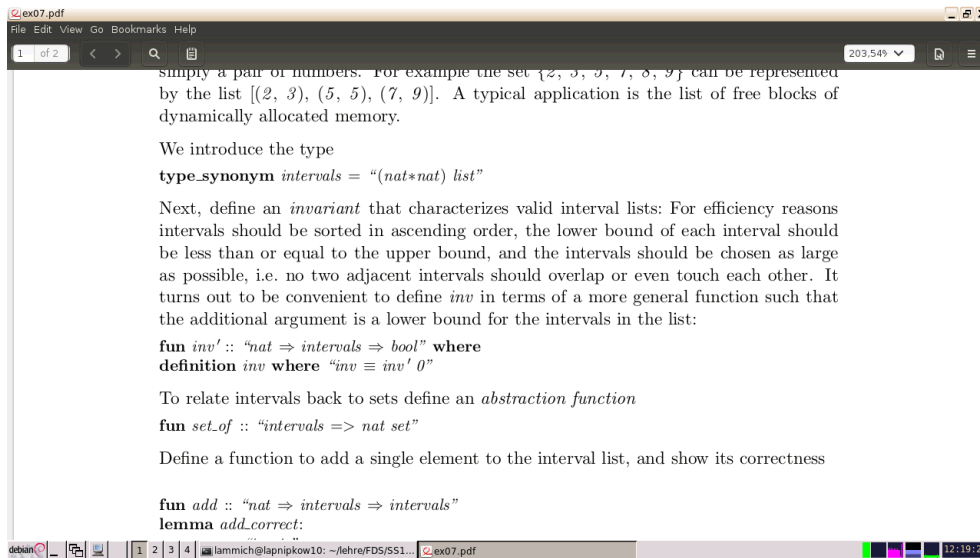
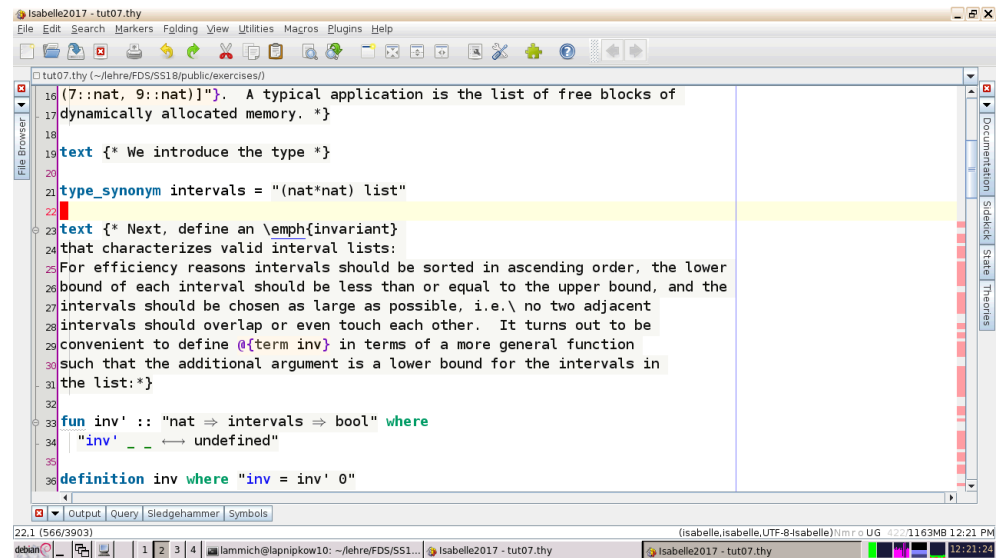
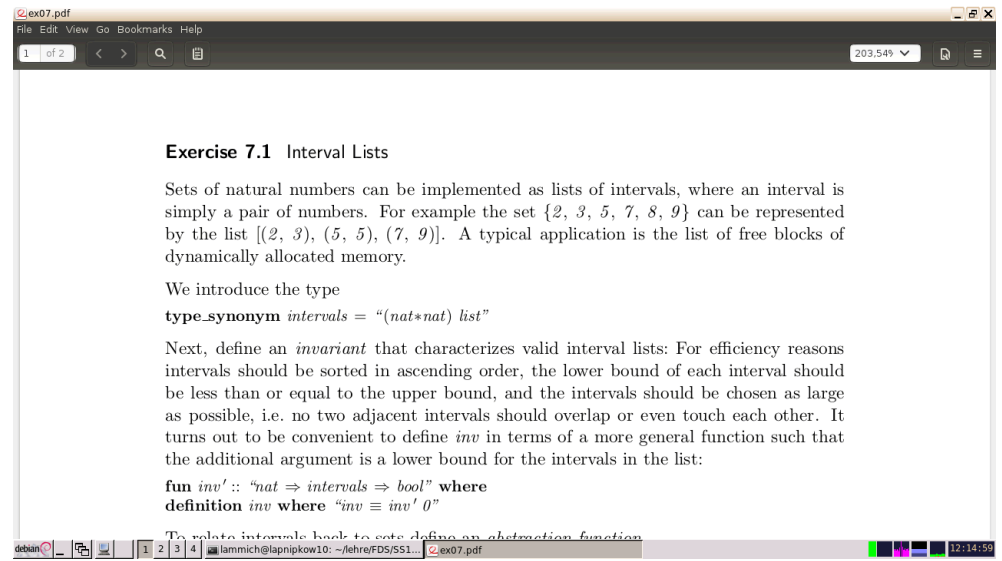
Script generated by TTT

Title: Lammich: FDS Tutorial (25.05.2018)

Date: Fri May 25 12:15:00 CEST 2018

Duration: 97:01 min

Pages: 112



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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
21 type_synonym intervals = "(nat*nat) list"
22
23 text {* Next, define an \emph{invariant}
24 that characterizes valid interval lists:
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{\term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list: *}
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> undefined"
35
36 definition inv where "inv = inv' 0"
37
38
39
40 text {* To relate intervals back to sets define an \emph{abstraction function} *}
41
Output Query Sledgehammer Symbols
34.25 (1172/3904) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 55/01163MB 12:22 PM

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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
21 type_synonym intervals = "(nat*nat) list"
22
23 text {* Next, define an \emph{invariant}
24 that characterizes valid interval lists:
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{\term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list: *}
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> True"
35   "inv' n (()) <-> True"
36
37 definition inv where "inv = inv' 0"
38
39
40 text {* To relate intervals back to sets define an \emph{abstraction function} *}
41
Output Query Sledgehammer Symbols
35.14 (1182/3915) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 69/01153MB 12:22 PM

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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
21 type_synonym intervals = "(nat*nat) list"
22
23 text {* Next, define an \emph{invariant}
24 that characterizes valid interval lists:
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{\term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list: *}
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> True"
35   "inv' n ((a,b)#ivs) <-> True"
36
37 definition inv where "inv = inv' 0"
38
39
40 text {* To relate intervals back to sets define an \emph{abstraction function} *}
41
Output Query Sledgehammer Symbols
35.25 (1193/3926) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 117/1164MB 12:23 PM

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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
21 type_synonym intervals = "(nat*nat) list"
22
23 text {* Next, define an \emph{invariant}
24 that characterizes valid interval lists:
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{\term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list: *}
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> True"
35   "inv' n ((a,b)#ivs) <-> n <= a"
36
37 definition inv where "inv = inv' 0"
38
39
40 text {* To relate intervals back to sets define an \emph{abstraction function} *}
41
Output Query Sledgehammer Symbols
35.27 (1195/3928) Matches line 43: fun set_of :: "intervals => nat set"
(Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 28/1164MB 12:23 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list:*)
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> True"
35 | "inv' n ((a,b)#ivs) <-> n <= a & a <= b & inv' (b+2) ivs"
36
37 definition inv where "inv = inv' 0"
38
39
const
inv' :: "nat => (nat x nat) list => bool"
Found termination order: "(λp. size_list (λp. size (snd p)) (snd p)) <*> mlex <*> {}"
Output Query Sledgehammer Symbols
35.52 (1.220/3951) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 43/1/322MB 12:24 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
25 For efficiency reasons intervals should be sorted in ascending order, the lower
26 bound of each interval should be less than or equal to the upper bound, and the
27 intervals should be chosen as large as possible, i.e.\ no two adjacent
28 intervals should overlap or even touch each other. It turns out to be
29 convenient to define @{term inv} in terms of a more general function
30 such that the additional argument is a lower bound for the intervals in
31 the list:*)
32
33 fun inv' :: "nat => intervals => bool" where
34   "inv' n [] <-> True"
35 | "inv' n ((a,b)#ivs) <-> n <= a & a <= b & inv' (b+2) ivs"
36
37 definition inv where "inv = inv' 0"
38
39
const
inv :: "(nat x nat) list => bool"
Output Query Sledgehammer Symbols
37.36 (1.257/3951) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 43/1/322MB 12:24 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
35 | "inv' n ((a,b)#ivs) <-> n <= a & a <= b & inv' (b+2) ivs"
36
37 definition inv where "inv = inv' 0"
38
39
40
41 text {* To relate intervals back to sets define an \emph{abstraction function} *}
42
43 fun set_of :: "intervals => nat set"
44 where
45   "set_of _ = undefined"
46
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
const
set_of :: "(nat x nat) list => nat set"
Found termination order: "{}"
Output Query Sledgehammer Symbols
42.1 (1.342/3951) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 43/1/322MB 12:24 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
36
37 definition inv where "inv = inv' 0"
38
39
40
41 text {* To relate intervals back to sets define an \emph{abstraction function} *}
42
43 fun set_of :: "intervals => nat set"
44 where
45   "set_of _ = undefined"
46
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
50
const
set_of :: "(nat x nat) list => nat set"
Found termination order: "{}"
Output Query Sledgehammer Symbols
46.1 (1.411/3951) (isabelle.isabelle.UTF-8-isabelle)Nmr o UG 43/1/322MB 12:25 PM

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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
36 definition inv where "inv = inv' 0"
37
38
39
40
41 text {* To relate intervals back to sets define an \emph{abstraction function} *}
42
43 fun set_of :: "intervals => nat set"
44 where
45   "set_of [] = {}"
46   | "set_of ((a,b)#ivs) = {}"
47
48 text <Define a function to add a single element to the interval list,
49 and show its correctness>
50
Malformed command syntax
46.27 (1.430/3972) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 69/1.325MB 12:26 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
39
40
41 text {* To relate intervals back to sets define an \emph{abstraction function} *}
42
43 fun set_of :: "intervals => nat set"
44 where
45   "set_of [] = {}"
46   | "set_of ((a,b)#ivs) = {a..b} U set_of ivs"
47
48 text <Define a function to add a single element to the interval list,
49 and show its correctness>
50
51
52 fun add :: "nat => intervals => intervals"
53 where
54
50.1 (1.549/3990) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 55/1.325MB 12:27 PM

```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
43 fun set_of :: "intervals => nat set"
44 where
45   "set_of [] = {}"
46   | "set_of ((a,b)#ivs) = {a..b} U set_of ivs"
47
48 text <Define a function to add a single element to the interval list,
49 and show its correctness>
50
51
52 fun add :: "nat => intervals => intervals"
53 where
54   "add _ _ = undefined"
55
56 Lemma add_correct:
57   assumes "inv is"
54.1 (1.600/3990) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 69/1.325MB 12:27 PM

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Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
45 "set_of [] = {}"
46 "set_of ((a,b)#ivs) = {a..b} U set_of ivs"
47
48 text <Define a function to add a single element to the interval list,
49 and show its correctness>
50
51
52 fun add :: "nat => intervals => intervals"
53 where
54   "add i [] = [(i,i)]"
55   "add i ((a,b)#ivs) = (i)"
56
57 Lemma add_correct:
58   assumes "inv is"
59   shows "inv (add x is) = insert x (set_of is)"
55.25 (1.647/4016) (Isabelle, Isabelle, UTF-8-Isabelle) Nm r o UG 10/1.301MB 12:32 PM

```

Inner syntax error: unexpected end of input
Failed to parse prop

Isabelle2017 - tut07.thy (modified)

```
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
50
51 fun add :: "nat => intervals => intervals"
52 where
53 "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   if i+1 < a then
56     (i,i)#(a,b)#ivs
57   else
58     (i,i)#ivs
59 )"
60
61 lemma add_correct:
```

Inner syntax error
Failed to parse prop

56.19 (1.666/4040) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 511/1301MB 12:32 PM

Isabelle2017 - tut07.thy (modified)

```
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
50
51 fun add :: "nat => intervals => intervals"
52 where
53 "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   if i+1 < a then (i,i)#(a,b)#ivs
56   else if i+1 = a then (i,i)#ivs
57   else
58     (i,i)#ivs
59 )"
60
61 lemma add_correct:
```

Inner syntax error
Failed to parse prop

57.28 (1.711/4085) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 779/1301MB 12:33 PM

Isabelle2017 - tut07.thy (modified)

```
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
50
51 fun add :: "nat => intervals => intervals"
52 where
53 "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   if i+1 < a then (i,i)#(a,b)#ivs
56   else if i+1 = a then (i,b)#ivs
57   else if i < b then (a,b)#ivs
58   else
59     (i,i)#ivs
60 )"
61 lemma add_correct:
```

Inner syntax error
Failed to parse prop

58.27 (1.745/4119) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 58/1324MB 12:34 PM

Isabelle2017 - tut07.thy (modified)

```
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
47 text <Define a function to add a single element to the interval list,
48 and show its correctness>
49
50
51 fun add :: "nat => intervals => intervals"
52 where
53 "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   if i+1 < a then (i,i)#(a,b)#ivs
56   else if i+1 = a then (i,b)#ivs
57   else if i < b then (a,b)#ivs
58   else if i=b+1 then (i,i)#ivs
59   else
60     (i,i)#ivs
61 )"
62 lemma add_correct:
```

Inner syntax error
Failed to parse prop

59.24 (1.773/4147) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 2/1324MB 12:35 PM

Isabelle2017 - tut07.thy (modified)

```
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
text <Define a function to add a single element to the interval list,
and show its correctness>

fun add :: "nat => intervals => intervals"
where
  "add i [] = [(i,i)]"
| "add i ((a,b)#ivs) = (
  if i+1 < a then (i,i)#(a,b)#ivs
  else if i+1 = a then (i,b)#ivs
  else if i < b then (a,b)#ivs
  else if i=b+1 then case
  )"
```

Inner syntax errors
Failed to parse prop

59.29 (1.778/4152) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 310/1324MB 12:36 PM

Isabelle2017 - tut07.thy

```
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
51 fun add :: "nat => intervals => intervals"
52 where
53   "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   | if i+1 < a then (i,i)#(a,b)#ivs
56   | else if i+1 = a then (i,b)#ivs
57   | else if i < b then (a,b)#ivs
58   | else if i=b+1 then case ivs of
59     [] => [(a,i)]
60     | (c,d)#ivs' => if i + 1 = c then (a,d)#ivs' else (a,i)#(c,d)#ivs'
61     else (a,b)#add i ivs
62   )"
63
64
65 Lemma add_correct:
```

57.1 (1.684/4274) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 100/1324MB 12:39 PM

Isabelle2017 - tut07.thy

```
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
51 fun add :: "nat => intervals => intervals"
52 where
53   "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   | if i+1 < a then (i,i)#(a,b)#ivs
56   | else if i+1 = a then (i,b)#ivs
57   | else if i < b then (a,b)#ivs
58   | else if i=b+1 then case ivs of
59     [] => [(a,i)]
60     | (c,d)#ivs' => if i + 1 = c then (a,d)#ivs' else (a,i)#(c,d)#ivs'
61     else (a,b)#add i ivs
62   )"
63
64
65 Lemma add_correct:
```

consts
add :: "nat => (nat × nat) list => (nat × nat) list"
Found termination order: "(λp. size_list (λp. size (snd p)) (snd p)) <*mlex*> {}"

59.32 (1.781/4274) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 107/1324MB 12:39 PM

Isabelle2017 - tut07.thy

```
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
51 fun add :: "nat => intervals => intervals"
52 where
53   "add i [] = [(i,i)]"
54 | "add i ((a,b)#ivs) = (
55   | if i+1 < a then (i,i)#(a,b)#ivs
56   | else if i+1 = a then (i,b)#ivs
57   | else if i < b then (a,b)#ivs
58   | else if i=b+1 then case ivs of
59     [] => [(a,i)]
60     | (c,d)#ivs' => if i + 1 = c then (a,d)#ivs' else (a,i)#(c,d)#ivs'
61     else (a,b)#add i ivs
62   )"
63
64
65 Lemma add_correct:
```

consts
add :: "nat => (nat × nat) list => (nat × nat) list"
Found termination order: "(λp. size_list (λp. size (snd p)) (snd p)) <*mlex*> {}"

59.25 (1.774/4274) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 122/1324MB 12:40 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
51 fun add :: "nat ⇒ intervals ⇒ intervals"
52 where
53 "add i [] = [(i,i)]"
54 "add i ((a,b)#ivs) = (
55 | if i+1 < a then (i,i)#(a,b)#ivs
56 | else if i+1 = a then (i,b)#ivs
57 | else if i ≤ b then (a,b)#ivs
58 | else if i=b+1 then case ivs of
59 | [] ⇒ [(a,i)]
60 | (c,d)#ivs' ⇒ if i + 1 = c then (a,d)#ivs' else (a,i)#(c,d)#ivs'
61 | else (a,b)#add i ivs
62 )"
63
64
65 lemma add_correct:
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tut07.thy (~/lehre/FDS/SS18/public/exercises/)
53 | "merge_aux a b [] = [(a,b)]"
54 | "merge_aux a b ((c,d)#ivs) = (if b+1<c then (a,d)#ivs else (a,b)#(c,d)#ivs)"
55
56 fun add :: "nat => intervals => intervals"
57 where
58   "add i [] = [(i,i)]"
59 | "add i ((a,b)#ivs) = (
60   if i+1 < a then (i,i)#(a,b)#ivs
61   else if i+1 = a then (i,b)#ivs
62   else if i < b then (a,b)#ivs
63   else if i=b+1 then merge_aux a i ivs
64   else (a,b)#add i ivs
65 )"
66
67 lemma add_correct:
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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
70 Lemma add_pres_inv: "inv is  $\implies$  inv (add x is)"
71
72
73 Lemma add_correct:
74   assumes "inv is"
75   shows "inv (add x is)" "set_of (add x is) = {x}  $\cup$  set_of is"
76   oops
77
78 text <Hints:
79   ■ Sketch the different cases (position of element relative to the first interval of the list)
80   on paper first
81   ■ In one case, you will also need information about the second interval of the list.
82   Do this case split via an auxiliary function! Otherwise, you may end up with a recursion equation of t

```

```

proof (prove)
goal (1 subgoal):
1. inv is  $\implies$  inv (add x is)

```

70.39 (2015/4385) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 16/1328MB 12:47 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 12:47:09

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
68
69
70 Lemma add_pres_inv: "inv is  $\implies$  inv (add x is)"
71
72
73 Lemma add_correct:
74   assumes "inv is"
75   shows "inv (add x is)" "set_of (add x is) = {x}  $\cup$  set_of is"
76   oops
77
78 text <Hints:
79   ■ Sketch the different cases (position of element relative to the first interval of the list)
80   on paper first

```

```

proof (prove)
goal (1 subgoal):
1. inv is  $\implies$  inv (add x is)

```

72.1 (2024/4385) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 21/1328MB 12:47 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 12:47:25

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64
65   else if i<b then (a,b)#ivs
66   else if i=b+1 then merge_aux a i ivs
67   else (a,b)#add i ivs
68   )"
69
70 Lemma add_pres_inv: "inv is  $\implies$  inv (add x is)"
71   unfolding inv_def
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x}  $\cup$  set_of is"

```

```

proof (prove)
goal (1 subgoal):
1. inv ' 0 is  $\implies$  inv ' 0 (add x is)

```

71.20 (2042/4406) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 16/1328MB 12:48 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) 12:48:00

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64
65   else if i<b then (a,b)#ivs
66   else if i=b+1 then merge_aux a i ivs
67   else (a,b)#add i ivs
68   )"
69
70 Lemma add_pres_inv: "inv is  $\implies$  inv (add x is)"
71   unfolding inv_def.
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x}  $\cup$  set_of is"

```

```

proof (prove)
goal (1 subgoal):
1. inv is  $\implies$  inv (add x is)

```

70.21 (1997/4406) (isabelle.isabelle.UTF-8-Isabelle)Nmr o UG 16/1328MB 12:48 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) 12:48:15

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64   else if i<b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n<x ==> inv' n is ==> inv' n (add x is)"
71   unfolding inv_def
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"

proof (prove)
goal (1 subgoal):
1. [n < x; inv' n is] ==> inv' n (add x is)

```

70.58 (2034/4417) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 740/1 28MB 12:49 PM

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64   else if i<b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n<x ==> inv' n is ==> inv' n (add x is)"
71   unfolding inv_def
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"

proof (prove)
goal (1 subgoal):
1. [n < x; inv' n is] ==> inv' n (add x is)

```

70.44 (2020/4417) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 754/1 28MB 12:51 PM

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Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64   else if i<b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n<x ==> inv' n ivs ==> inv' n (add x ivs)"
71   unfolding inv_def
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"

proof (prove)
goal (1 subgoal):
1. [n < x; inv' n ivs] ==> inv' n (add x ivs)

```

71.20 (2056/4419) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 852/1 8MB 12:51 PM

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
64   else if i<b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n<x ==> inv' n ivs ==> inv' n (add x ivs)"
71   apply (induction ivs)
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"

proof (prove)
goal (2 subgoals):
1. [n < x; inv' n []] ==> inv' n (add x [])
2. ^a ivs.
[[n < x; inv' n ivs] ==> inv' n (add x ivs); n < x; inv' n (a # ivs)] ==> inv' n (add x (a # ivs))

```

71.23 (2059/4423) (isabelle,isabelle,UTF-8-Isabelle)Nmr o UG 954/1 327MB 12:53 PM

Isabelle2017 - tut07.thy

```

64   else if i≤b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n≤x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs)
72
73
74 Lemma add_correct:
75   assumes "inv is"
76   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"

```

Proof state: Auto update: Update: Search: 100%

```

proof (prove)
goal (2 subgoals):
1. [n ≤ x; inv' n []] ⇒ inv' n (add x [])
2. [∧ a ivs.
   [[n ≤ x; inv' n ivs] ⇒ inv' n (add x ivs); n ≤ x; inv' n (a # ivs)] ⇒ inv' n (add x (a # ivs))]

```

71.23 (2059/4423) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 38.1/1327MB 12:55 PM

Isabelle2017 - tut07.thy

```

64   else if i≤b then (a,b)#ivs
65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n≤x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72
73
74 Lemma add_correct:

```

Proof state: Auto update: Update: Search: 100%

```

proof (prove)
goal (2 subgoals):
1. [∧ n. [n ≤ x; inv' n []] ⇒ inv' n (add x [])]
2. [∧ a ivs n.
   [[∧ n. [n ≤ x; inv' n ivs] ⇒ inv' n (add x ivs); n ≤ x; inv' n (a # ivs)] ⇒ inv' n (add x (a # ivs))]

```

71.36 (2072/4436) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 50.1/1327MB 12:55 PM

Isabelle2017 - tut07.thy (modified)

```

65   else if i=b+1 then merge_aux a i ivs
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n≤x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply simp
74
75 Lemma add_correct:

```

Proof state: Auto update: Update: Search: 100%

```

proof (prove)
goal (1 subgoal):
1. [∧ a b ivs n.
   [[∧ n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
   x = Suc b]
   ⇒ inv' n (merge_aux a (Suc b) ivs)]

```

72.10 (2083/4446) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 69.0/1327MB 12:56 PM

Isabelle2017 - tut07.thy

```

67   )"
68
69
70 Lemma add_pres_inv: "n≤x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply simp
74
75 Lemma add_correct:
76   assumes "inv is"
77

```

Proof state: Auto update: Update: Search: 100%

```

proof (prove)
goal (1 subgoal):
1. [∧ a ivs n.
   [[∧ n. [n ≤ x; inv' n ivs] ⇒ inv' n (add x ivs); n ≤ x; inv' n (a # ivs)] ⇒ inv' n (add x (a # ivs))]

```

72.13 (2086/4462) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 70.0/1327MB 12:57 PM

```
Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
67 |)"
68
69
70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply cllarsimp
74
75
76 Lemma add_correct:
77 assumes "inv is"

proof (prove)
goal (1 subgoal):
1. ∧ a b ivs n.
  [∧ n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)

Output Query Sledgehammer Symbols
73.17 (2103/4462) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 855/1327MB 12:57 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) Isabelle2017 - tut07.thy (modified) 12:57:36
```

```
Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
67 |)"
68
69
70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply auto
74
75
76 Lemma add_correct:
77 assumes "inv is"

proof (prove)
goal (1 subgoal):
1. ∧ a b ivs n.
  [∧ n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)

Output Query Sledgehammer Symbols
73.13 (2099/4462) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 150/1330MB 12:58 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) Isabelle2017 - tut07.thy (modified) 12:58:00
```

```
Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
66 else (a,b)#add i ivs
67 |)"
68
69
70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply auto
74
75
76 Lemma add_correct:

proof (prove)
goal (1 subgoal):
1. ∧ a b ivs n.
  [∧ n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)

Output Query Sledgehammer Symbols
73.13 (2099/4462) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 44/1330MB 12:58 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) Isabelle2017 - tut07.thy (modified) 12:58:36
```

```
Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
66 else (a,b)#add i ivs
67 |)"
68
69
70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply simp
73 apply auto
74
75
76 Lemma add_correct:

proof (prove)
goal (1 subgoal):
1. ∧ a b ivs n.
  [∧ n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)

Output Query Sledgehammer Symbols
72.1 (2074/4462) (Isabelle, Isabelle, UTF-8-Isabelle) Nmr o UG 17/1330MB 12:59 PM
debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) Isabelle2017 - tut07.thy (modified) 12:59:01
```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n <= x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71   apply (induction ivs arbitrary: n)
72   apply auto
73
74
75 Lemma add_correct:
76   assumes "inv is"

proof (prove)
goal (1 subgoal):
1. ∀a b ivs n.
  [[∧n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs);
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)
  
```

73.1 (2087/4449) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 1/1330MB 1:00 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:00:02

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n <= x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71   apply (induction ivs arbitrary: n)
72   apply auto
73
74
75 Lemma add_correct:
76   assumes "inv is"

consts
add :: "nat ⇒ (nat × nat) list ⇒ (nat × nat) list"
Found termination order: "(λp. size_list (λp. size (snd p)) (snd p)) < *mlex* > {}"
  
```

69.1 (1976/4449) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 4/1330MB 1:00 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:00:23

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
52
53
54 fun merge_aux where
55   "merge_aux a b [] = [(a,b)]"
56 | "merge_aux a b ((c,d)#ivs) = (if b+1=c then (a,d)#ivs else (a,b)#(c,d)#ivs)"
57
58 fun add :: "nat ⇒ intervals ⇒ intervals"
59 where
60   "add i [] = [(i,i)]"
61 | "add i ((a,b)#ivs) = (
62   if i+1 < a then (i,i)#(a,b)#ivs
  
```

59.1 (1746/4449) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 1/1330MB 1:01 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:01:05

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
66   else (a,b)#add i ivs
67   )"
68
69
70 Lemma add_pres_inv: "n <= x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71   apply (induction ivs arbitrary: n)
72   apply auto
73   a
74
75
76 Lemma add_correct:

proof (prove)
goal (1 subgoal):
1. ∀a b ivs n.
  [[∧n. [n ≤ Suc b; inv' n ivs] ⇒ inv' n (add (Suc b) ivs); n ≤ a; a ≤ b; inv' (Suc (Suc b)) ivs;
  x = Suc b]
  ⇒ inv' n (merge_aux a (Suc b) ivs)
  
```

73.4 (2090/4453) (isabelle,isabelle,UTF-8-isabelle)tmr o UG 1/1330MB 1:01 PM
 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy (modified) 13:01:31

Isabelle2017 - tut07.thy (modified)

```

70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 apply (induction ivs arbitrary: n)
72 apply auto
73 apply (cases_tac ivs)
74 apply auto
75
76 Lemma add_correct:

```

proof (prove)
goal (2 subgoals):
1. $\bigwedge a b \text{ ivs } n. [\bigwedge n. [n \leq \text{Suc } b; \text{inv}' n \text{ ivs}] \Rightarrow \text{inv}' n (\text{add } (\text{Suc } b) \text{ ivs}); n \leq a; a \leq b; \text{inv}' (\text{Suc } (\text{Suc } b)) \text{ ivs}; x = \text{Suc } b; \text{ivs} = []] \Rightarrow \text{inv}' n (\text{merge_aux } a (\text{Suc } b) \text{ ivs})$
2. $\bigwedge a b \text{ ivs } n \text{ aa list.}$

73.13 (2099/4485) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 771/1330MB 1:02 PM

Isabelle2017 - tut07.thy (modified)

```

70 Lemma add_pres_inv: "n ≤ x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
71 proof (induction ivs arbitrary: n)
72 case Nil
73 then show ?case by auto
74 next
75 case (Cons a ivs)
76 then show ?case apply auto
77 qed
78 apply auto
79
80

```

Failed to apply proof method:
using this:

- $[?n \leq x; \text{inv}' ?n \text{ ivs}] \Rightarrow \text{inv}' ?n (\text{add } x \text{ ivs})$
- $n \leq x$
- $\text{inv}' n (a \# \text{ ivs})$

goal (1 subgoal):
1. $\text{inv}' n (\text{add } x (a \# \text{ ivs}))$

76.29 (2162/4542) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 341/329MB 1:02 PM

Isabelle2017 - tut07.thy

```

73 proof (induction ivs arbitrary: n)
74 case Nil
75 then show ?case by auto
76 next
77 case (Cons a ivs)
78 then show ?case apply auto
79 qed
80 apply auto
81
82
83 Lemma add_correct:

```

proof (state)
this:

- $[?n \leq x; \text{inv}' ?n \text{ ivs}] \Rightarrow \text{inv}' ?n (\text{add } x \text{ ivs})$
- $n \leq x$
- $\text{inv}' n (a \# \text{ ivs})$

goal (1 subgoal):

78.19 (2182/4572) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 99/1332MB 1:03 PM

Isabelle2017 - tut07.thy

```

75 then show ?case by auto
76 next
77 case (Cons a ivs)
78 then show ?case
79 apply (cases a)
80 apply auto
81 apply (cases ivs)
82 apply auto
83 done
84 qed
85
86

```

proof (prove)
goal:
No subgoals!

81.1 (2217/4629) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 71/332MB 1:05 PM

Isabelle2017 - tut07.thy

```

75 then show ?case by auto
76 next
77 case (Cons a ivs)
78 then show ?case
79   apply (cases a)
80   apply auto
81   apply (cases ivs)
82   apply auto
83   done
84 qed

```

80.1 (2202/4629) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 132/1332MB 1:07 PM

```

proof (prove)
goal (1 subgoal):
1.  $\wedge aa b. [\wedge n. [n \leq \text{Suc } b; \text{inv}' n \text{ ivs}] \Rightarrow \text{inv}' n (\text{add } (\text{Suc } b) \text{ ivs}); a = (aa, b); n \leq aa; aa \leq b;$ 
 $\text{inv}' (\text{Suc } (\text{Suc } b)) \text{ ivs}; x = \text{Suc } b]$ 
 $\Rightarrow \text{inv}' n (\text{merge\_aux } aa (\text{Suc } b) \text{ ivs})$ 

```

13:07:18

Isabelle2017 - tut07.thy

```

75 then show ?case by auto
76 next
77 case (Cons a ivs)
78 then show ?case
79   apply (cases a)
80   apply (cases ivs)
81   apply auto
82   done
83 qed

```

82.1 (2239/4614) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 132/1332MB 1:07 PM

```

proof (prove)
goal:
No subgoals!

```

13:07:34

Isabelle2017 - tut07.thy

```

72 using assms
73 proof (induction ivs arbitrary: n)
74 case Nil
75 then show ?case by auto
76 next
77 case (Cons a ivs)
78 then show ?case
79   apply (cases a)
80   apply (cases ivs)
81   apply auto
82   done
83 qed

```

76.1 (2139/4614) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 132/1332MB 1:08 PM

```

proof (state)
goal (1 subgoal):
1.  $\wedge a \text{ ivs } n. [\wedge n. [n \leq x; \text{inv}' n \text{ ivs}] \Rightarrow \text{inv}' n (\text{add } x \text{ ivs}); n \leq x; \text{inv}' n (a \# \text{ ivs})]$ 
 $\Rightarrow \text{inv}' n (\text{add } x (a \# \text{ ivs}))$ 

```

13:08:03

Isabelle2017 - tut07.thy

```

68
69
70 Lemma add_pres_inv:
71 " $n \leq x \Rightarrow \text{inv}' n \text{ ivs} \Rightarrow \text{inv}' n (\text{add } x \text{ ivs})"$ 
72 proof (induction ivs arbitrary: n)
73 case Nil
74 then show ?case by auto
75 next
76 case (Cons a ivs)
77 then show ?case
78   apply (cases a)
79   apply (cases ivs)

```

71.1 (1997/4634) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 132/1332MB 1:09 PM

```

proof (prove)
goal (1 subgoal):
1.  $[n \leq x; \text{inv}' n \text{ ivs}] \Rightarrow \text{inv}' n (\text{add } x \text{ ivs})$ 

```

13:09:40

Isabelle2017 - tut07.thy

```
70 Lemma add_pres_inv:
71   "n < x ⇒ inv' n ivs ⇒ inv' n (add x ivs)"
72 proof (induction ivs arbitrary: n)
73   case Nil
74   then show ?case by auto
75 next
76   case (Cons a ivs)
77   then show ?case
78     apply (cases a)
```

proof (prove)
goal (1 subgoal):
1. [n < x; inv' n ivs] ⇒ inv' n (add x ivs)

Isabelle2017 - tut07.thy

```
82 qed
83
84 Lemma set_of_add:
85   assumes "inv is"
86   shows "set_of (add x is) = {x} ∪ set_of is"
87   using assms
88   unfolding inv_def
89
90 Lemma add_correct:
91   assumes "inv is"
92   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"
93   oops
```

proof (prove)
using this:
inv is
goal (1 subgoal):
1. set_of (add x is) = {x} ∪ set_of is

Isabelle2017 - tut07.thy

```
82 qed
83
84 Lemma set_of_add:
85   assumes "n < x"
86   assumes "inv' n is"
87   shows "set_of (add x is) = {x} ∪ set_of is"
88   using assms
89   unfolding inv_def
90
91 Lemma add_correct:
```

proof (prove)
using this:
n < x
inv' n is
goal (1 subgoal):
1. set_of (add x is) = {x} ∪ set_of is

Isabelle2017 - tut07.thy

```
90 case Nil
91 then show ?case by auto
92 next
93 case (Cons a ivs)
94 then show ?case
95   apply (cases a)
96   apply (cases ivs)
```

proof (state)
this:
[?n < x; inv' ?n ivs] ⇒ set_of (add x ivs) = {x} ∪ set_of ivs
n < x
inv' n (a # ivs)
goal (1 subgoal):
1. ∧ a ivs n.
[∧ n. [n < x; inv' n ivs] ⇒ set_of (add x ivs) = {x} ∪ set_of ivs; n < x; inv' n (a # ivs)]
⇒ set_of (add x (a # ivs)) = {x} ∪ set_of (a # ivs)

Isabelle2017 - tut07.thy

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_splits)
done
qed

```

theorem set_of_add:
 $[?n \leq ?x; \text{inv } ?n \text{ ivs}] \Rightarrow \text{set_of (add ?x ?ivs)} = \{?x\} \cup \text{set_of ?ivs}$

98.2 (2536/4911) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1:19MB 1:15 PM

Isabelle2017 - tut07.thy (modified)

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_split_asm)
done
qed

```

proof (prove)
goal (10 subgoals):
1. $\bigwedge a b \text{ aaa } ba \text{ list } xa. \llbracket n. n \leq \text{Suc } b \Rightarrow \text{set_of (if Suc (Suc } b) < \text{aaa then (Suc } b, \text{Suc } b) \# (\text{aaa}, ba) \# \text{list else if Suc } b + 1 = \text{aaa then (Suc } b, ba) \# \text{list else if Suc } b \leq ba \text{ then (aaa}, ba) \# \text{list else if Suc } b = ba + 1 \text{ then merge_aux aaa (Suc } b) \text{list else (aaa}, ba) \# \text{add (Suc } b) \text{list} = \text{insert (Suc } b) (\text{aaa}.ba) \cup \text{set_of list}; a = (aa, b); ivs = (aaa, ba) \# \text{list}; } n \leq aa; aa \leq b; x = \text{Suc } b;$

97.36 (2536/4914) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1:19MB 1:15 PM

Isabelle2017 - tut07.thy (modified)

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_split_asm)
done
qed

```

proof (prove)
goal:
No subgoals!

97.36 (2536/4914) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1:19MB 1:16 PM

Isabelle2017 - tut07.thy (modified)

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_splits)
done
qed

```

proof (prove)
goal:
No subgoals!

97.33 (2533/4911) Input/output complete (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1:19MB 1:16 PM

Isabelle2017 - tut07.thy

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_splits)
done
qed

```

proof (prove)
goal:
No subgoals!

98.9 (2543/4911) (isabelle,isabelle,UTF-8-Isabelle)nmro UG 709/1198MB 1:16 PM

Isabelle2017 - tut07.thy

```

then show ?case
  apply (cases a)
  apply (cases ivs)
  apply (auto split: if_splits)
done
qed

```

Lemma add_correct:
assumes "inv is"
shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
oops

proof (prove)
goal:
No subgoals!

98.1 (2535/4911) (isabelle,isabelle,UTF-8-Isabelle)nmro UG 736/1198MB 1:16 PM

Isabelle2017 - tut07.thy

```

done
qed

```

Lemma add_correct:
assumes "inv is"
shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
using add_pres_inv assms tut07.inv_def apply fastforce
sledgehammer

text <Hints:

Sledgehammering...
Proof found...
"e": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (100 ms)
"z3": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (56 ms)
"cvc4": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (44 ms)
"pvs": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (30 ms)

106.15 (2666/4977) (isabelle,isabelle,UTF-8-Isabelle)nmro UG 517/1174MB 1:17 PM

Isabelle2017 - tut07.thy

```

done
qed

```

Lemma add_correct:
assumes "inv is"
shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
using add_pres_inv assms tut07.inv_def apply fastforce
sledgehammer

text <Hints:

Sledgehammering...
Proof found...
"e": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (100 ms)
"z3": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (56 ms)
"cvc4": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (44 ms)
"pvs": Try this: using add_pres_inv assms tut07.inv_def apply fastforce (30 ms)

107.15 (2723/4977) (isabelle,isabelle,UTF-8-Isabelle)nmro UG 520/1174MB 1:18 PM

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
done
qed
103
104 lemma add_correct:
105   assumes "inv is"
106   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
107   using add_pres_inv assms tut07.inv_def apply fastforce
108   sledgehammer
109
110 text <Hints:
111
112
Sledgehammering...
Output Query Sledgehammer Symbols
107.15 (2723/4977) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1065/174MB 1:18 PM

```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
done
qed
103
104 lemma add_correct:
105   assumes "inv is"
106   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
107   using add_pres_inv assms tut07.inv_def apply fastforce
108   sledgehammer
109
110 text <Hints:
111
112
proof (prove)
using this:
  • [?n ≤ ?x; inv' ?n ?ivs] ⇒ inv' ?n (add ?x ?ivs)
  • inv is
  • inv = inv' 0
Output Query Sledgehammer Symbols
106.15 (2666/4977) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1065/174MB 1:18 PM

```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
done
qed
103
104 lemma add_correct:
105   assumes "inv is"
106   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
107   using add_pres_inv assms tut07.inv_def apply fastforce
108   sledgehammer
109
110 text <Hints:
111   • Sketch the different cases (position of element relative to the first interval of the list)
112   on paper first
113
proof (prove)
goal (1 subgoal):
1. set_of (add x is) = {x} U set_of is
Output Query Sledgehammer Symbols
104.19 (2598/4977) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 1065/174MB 1:18 PM

```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
done
qed
103
104 lemma add_correct:
105   assumes "inv is"
106   shows "inv (add x is)" "set_of (add x is) = {x} U set_of is"
107   using add_pres_inv assms tut07.inv_def apply fastforce
108   using assms set_of_add tut07.inv_def by fastforce
109
110 text <Hints:
111   • Sketch the different cases (position of element relative to the first interval of the list)
112
theorem add_correct:
  • inv ?is ⇒ inv (add ?x ?is)
  • inv ?is ⇒ set_of (add ?x ?is) = {?x} U set_of ?is
Output Query Sledgehammer Symbols
108.1 (2761/5014) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 2/1153MB 1:20 PM

```

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
96 apply (cases ivs)
97 apply (auto split: if_splits)
98 done
99
100
101
102
103 Lemma add_correct:
104   assumes "inv is"
105   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"
106   using add_pres_inv assms tut07.inv_def apply fastforce
107   using assms set_of_add tut07.inv_def by fastforce
108
109
110
111
112
113
114

proof (prove)
goal (1 subgoal):
1. set_of (add x is) = {x} ∪ set_of is

```

99.1 (2544/5014) (isabelle.isabelle.UTF-8-isabelle)lmmr UG 3/1153MB 1:20 PM
 13:20:10

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109
110
111
112
113 Lemma add_correct:
114   assumes "inv is"
115   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"

```

106.1 (2604/5131) (isabelle.isabelle.UTF-8-isabelle)lmmr UG 385/1116MB 1:21 PM
 13:21:43

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
100
101 consts
102   a :: "nat"
103   b :: "nat set"
104   c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109
110
111
112 Lemma add_correct:

```

109.1 (2665/5131) (isabelle.isabelle.UTF-8-isabelle)lmmr UG 423/1116MB 1:22 PM
 13:22:00

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
100
101 consts
102   a :: "nat"
103   b :: "nat set"
104   c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109   apply (simp add: A)
110
111
112 Lemma add_correct:

```

109.21 (2685/5151) Input/output complete (isabelle.isabelle.UTF-8-isabelle)lmmr UG 542/1116MB 1:22 PM
 13:22:14

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109 apply (simp add: A)
110 using A apply simp
111
112
113 Lemma add_correct:
114 assumes "inv is"

proof (prove)
goal (1 subgoal):
1. insert a (b ∪ d) = c ∪ d

```

109.22 (2686/5173) (isabelle,isabelle,UTF-8-Isabelle)tmr o UG 1 69/1 097MB 1:23 PM

```

Isabelle2017 - tut07.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109 apply (simp add: A)
110 using A apply simp
111
112
113 Lemma add_correct:
114 assumes "inv is"

theorem A: {a} ∪ b = c

```

107.1 (2633/5173) (isabelle,isabelle,UTF-8-Isabelle)tmr o UG 1 72/1 097MB 1:23 PM

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109 apply (simp add: A)
110 using A apply simp
111
112
113 Lemma add_correct:
114 assumes "inv is"

proof (prove)
using this:
{a} ∪ b = c

goal (1 subgoal):
1. insert a (b ∪ d) = c ∪ d

```

110.9 (2695/5173) (isabelle,isabelle,UTF-8-Isabelle)tmr o UG 1 64/1 097MB 1:23 PM

```

Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109 apply (simp add: A)
110 using A apply simp
111
112

proof (prove)
goal (1 subgoal):
1. {a} ∪ b ∪ d = c ∪ d

```

108.23 (2656/5173) (isabelle,isabelle,UTF-8-Isabelle)tmr o UG 1 62/1 097MB 1:25 PM

Isabelle2017 - tut07.thy

```

104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109   apply (simp add: A)
110   using A apply simp
111
112
113 Lemma add_correct:
114   assumes "inv is"

```

proof (prove)
using this:
{a} ∪ b = c

goal (1 subgoal):
1. insert a (b ∪ d) = c ∪ d

110.3 (2689/5173) (isabelle.isabelle.UTF-8-isabelle)lmmr o UG 365/1080MB 1:26 PM

Isabelle2017 - tut07.thy (modified)

```

105 Lemma A: "{a} ∪ b = c" sorry
106
107 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
108   using A apply simp
109
110
111
112 Lemma add_correct:
113   assumes "inv is"
114   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"
115   using add_pres_inv assms tut07.inv_def apply fastforce

```

proof (prove)
goal:
No subgoals!

110.1 (2686/5151) Input/output complete (isabelle.isabelle.UTF-8-isabelle)lmmr o UG 545/1080MB 1:26 PM

Isabelle2017 - tut07.thy (modified)

```

102 a :: "nat"
103 b :: "nat set"
104 c :: "nat set"
105
106 Lemma A: "{a} ∪ b = c" sorry
107
108 Lemma B: "{a} ∪ b ∪ d = c ∪ d"
109   using A apply simp
110
111
112 Lemma add_correct:

```

proof (prove)
goal:
No subgoals!

105.1 (2603/5151) (isabelle.isabelle.UTF-8-isabelle)lmmr o UG 548/1080MB 1:26 PM

Isabelle2017 - tut07.thy

```

101 Lemma add_correct:
102   assumes "inv is"
103   shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"
104   using add_pres_inv assms tut07.inv_def apply fastforce
105   using assms set_of_add tut07.inv_def by fastforce
106
107
108
109 text <Hints:
110   ■ Sketch the different cases (position of element relative to the first interval of the list)
111   on paper first

```

theorem add_correct:
inv ?is ⇒ inv (add ?x ?is)
inv ?is ⇒ set_of (add ?x ?is) = {?x} ∪ set_of ?is

107.1 (2760/5013) (isabelle.isabelle.UTF-8-isabelle)lmmr o UG 99/1063MB 1:27 PM

Isabelle2017 - tut07.thy

```

shows "inv (add x is)" "set_of (add x is) = {x} ∪ set_of is"
using add_pres_inv assms tut07.inv_def apply fastforce
using assms set_of_add tut07.inv_def by fastforce

text <Hints:
  Sketch the different cases (position of element relative to the first interval of the list)
  on paper first
  In one case, you will also need information about the second interval of the list.
  Do this case split via an auxiliary function! Otherwise, you may end up with a recursion equation of t
  <f (x#xs) = ... case xs of x'#xs' => ... f (x'#xs') ...>

proof (prove)
goal (1 subgoal):
1. inv (add x is) &&& set_of (add x is) = {x} ∪ set_of is

```

110.1 (2775/5013)

Isabelle2017 - Sorting.thy

```

"merge [] ys = ys" |
"merge xs [] = xs" |
"merge (x#xs) (y#ys) = (if x ≤ y then x # merge xs (y#ys) else y # merge (x#xs) ys)"

fun msort :: "'a::linorder list ⇒ 'a list" where
"msort xs = (let n = length xs in
  if n ≤ 1 then xs
  else merge (msort (take (n div 2) xs)) (msort (drop (n div 2) xs)))"

declare msort.simps [simp del]

consts
msort :: "'a list ⇒ 'a list"
Found termination order: "length <math>{}</math>"

```

130.1 (3031/5664)

Isabelle2017 - Sorting.thy

```

"merge (x#xs) (y#ys) = (if x ≤ y then x # merge xs (y#ys) else y # merge (x#xs) ys)"

fun msort :: "'a::linorder list ⇒ 'a list" where
"msort xs = (let n = length xs in
  if n ≤ 1 then xs
  else merge (msort (take (n div 2) xs)) (msort (drop (n div 2) xs)))"

declare msort.simps [simp del]

(* We count the number of comparisons between list elements only *)

consts
msort :: "'a list ⇒ 'a list"
Found termination order: "length <math>{}</math>"

```

134.1 (3204/5664)

Isabelle2017 - tut07.thy

```

(* Optimized mergesort *)
fun msort2 :: "nat ⇒ 'a::linorder list ⇒ 'a list"
where "msort2 _ _ = undefined"

lemma "n = length xs ⇒ msort2 n xs = msort xs"
oops

text <Hint:
  Use @<math>{}</math> [source] msort.simps only when instantiated to a particular <math>{}</math>

proof (prove)
goal (1 subgoal):
1. n = length xs ⇒ msort2 n xs = msort xs

```

131.33 (3663/5013)

Isabelle2017 - tut07.thy

```

128 (* Optimized mergesort *)
129
130 fun msort2 :: "nat ⇒ 'a::linorder list ⇒ 'a list"
131 "msort2 n xs = (let n = length xs in
132  if n ≤ 1 then xs
133  else merge (msort2 ... (take (n div 2) xs)) (msort2 ... (drop (n div 2) xs)))"
134
135 Lemma "n = length xs ⇒ msort2 n xs = msort xs"
136
137 oops
138
139 text <Hint:
140

```

Outer syntax error: keyword "where" expected, but quoted string was found:
 "msort2 n xs = (let n = length xs in
 if n ≤ 1 then xs
 else merge (msort2 ... (take (n div 2) xs)) (msort2 ... (drop (n div 2) xs)))"

131.35 (3665/5113) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 98/1049MB 1:32 PM

Isabelle2017 - tut07.thy

```

130 fun msort2 :: "nat ⇒ 'a::linorder list ⇒ 'a list" where
131 "msort2 n xs = (
132  if n ≤ 1 then xs
133  else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 Lemma "n = length xs ⇒ msort2 n xs = msort xs"
136 oops
137
138 text <Hint:
139 Use @{thm [source] msort.simps} only when instantiated to a particular <xs>
140 (@{thm [source] msort.simps[of xs]}),

```

```

proof (prove)
goal (1 subgoal):
  1. n = length xs ⇒ msort2 n xs = msort xs
  linarith_split_limit exceeded (current value is 9)
  linarith_split_limit exceeded (current value is 9)
  linarith_split_limit exceeded (current value is 9)
  linarith_split_limit exceeded (current value is 9)
  linarith_split_limit exceeded (current value is 9)

```

135.22 (3792/5119) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 827/1049MB 1:35 PM

Isabelle2017 - tut07.thy (modified)

```

130 fun msort2 :: "nat ⇒ 'a::linorder list ⇒ 'a list" where
131 "msort2 n xs = (
132  if n ≤ 1 then xs
133  else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 Lemma "n = length xs ⇒ msort2 n xs = msort xs"
136
137
138 text <Hint:
139 Use @{thm [source] msort.simps} only when instantiated to a particular <xs>
140 (@{thm [source] msort.simps[of xs]}),

```

136.3 (3820/5115) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 845/1049MB 1:35 PM

Isabelle2017 - tut07.thy

```

128 (* Optimized mergesort *)
129
130 fun msort2 :: "nat ⇒ 'a::linorder list ⇒ 'a list" where
131 "msort2 n xs = (
132  if n ≤ 1 then xs
133  else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 Lemma "n = length xs ⇒ msort2 n xs = msort xs"
136 apply (induction n xs rule: msort2.induct)
137 apply auto
138

```

```

consts
msort2 :: "nat ⇒ 'a list ⇒ 'a list"
Found termination order: "(λp. size (fst p)) <+mlex+> {}"

```

133.11 (3683/5170) (isabelle.isabelle.UTF-8-isabelle)tmr o UG 811/1032MB 1:36 PM


```
Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
128 (* Optimized mergesort *)
129
130 fun msort2 :: "nat => 'a::linorder list => 'a list" where
131   "msort2 n xs = (
132     if n <= 1 then xs
133     else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 lemma "n = length xs ==> msort2 n xs = msort xs"
136   apply (induction n xs rule: msort2.induct)
137   apply auto
138
consts
msort2 :: "nat => 'a list => 'a list"
Found termination order: "(λp. size (fst p)) <+mlex+> {}"
[Output] [Query] [Sledgehammer] [Symbols]
133.2 (3674/5170) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 865.1 032MB 1:36 PM
deban [1] [2] [3] [4] iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:36:48
```

```
Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
128 (* Optimized mergesort *)
129
130 fun msort2 :: "nat => 'a::linorder list => 'a list" where
131   "msort2 n xs = (
132     if n <= 1 then xs
133     else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 lemma "n = length xs ==> msort2 n xs = msort xs"
136   apply (induction n xs rule: msort2.induct)
137   apply auto
138
consts
msort2 :: "nat => 'a list => 'a list"
Found termination order: "(λp. size (fst p)) <+mlex+> {}"
[Output] [Query] [Sledgehammer] [Symbols]
134.1 (3770/5170) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 892.1 032MB 1:37 PM
deban [1] [2] [3] [4] iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:37:15
```

```
Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
131 msort2 n xs = (
132   if n <= 1 then xs
133   else merge (msort2 (n div 2) (take (n div 2) xs)) (msort2 (n - n div 2) (drop (n div 2) xs)))"
134
135 declare msort2.simps[simp del]
136
137 lemma "n = length xs ==> msort2 n xs = msort xs"
138   apply (induction n xs rule: msort2.induct)
139   apply auto
140
141 text <Hint:
[Output] [Query] [Sledgehammer] [Symbols]
138.36 (3885/5202) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 906.1 1018MB 1:38 PM
deban [1] [2] [3] [4] iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:38:39

proof (prove)
goal (1 subgoal):
1. [¬ n ≤ 1; n div 2 = length (take (n div 2) xs)]
   => msort2 (n div 2) (take (n div 2) xs) = msort (take (n div 2) xs);
   [¬ n ≤ 1; n - n div 2 = length (drop (n div 2) xs)]
   => msort2 (n - n div 2) (drop (n div 2) xs) = msort (drop (n div 2) xs);
   n = length xs
   => msort2 n xs = msort xs
```

```
Isabelle2017 - tut07.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
tut07.thy (~/lehre/FDS/SS18/public/exercises/)
137 lemma "n = length xs ==> msort2 n xs = msort xs"
138 proof (induction n xs rule: msort2.induct)
139   case (1 n xs)
140   then show ?case
141     apply (auto simp: msort.simps[of xs] msort2.simps[of n xs])
142
143 qed
144
145 apply auto
146
147 text <Hint:
[Output] [Query] [Sledgehammer] [Symbols]
141.58 (3984/5304) (isabelle.isabelle,UTF-8-isabelle)tmr o UG 82.9 979MB 1:40 PM
deban [1] [2] [3] [4] iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy 13:40:58

proof (prove)
goal (2 subgoals):
1. [n = length xs; length xs ≤ Suc 0] ==> msort2 (length xs) xs = xs
2. [length xs div 2 = min (length xs) (length xs div 2) ==>
   msort2 (length xs div 2) (take (length xs div 2) xs) = msort (take (length xs div 2) xs);
   msort2 (length xs - length xs div 2) (drop (length xs div 2) xs) = msort (drop (length xs div 2) xs);
   n = length xs; ¬ length xs ≤ Suc 0]
   => msort2 (length xs) xs =
```

Isabelle2017 - tut07.thy (modified)

```

137 lemma "n = length xs  $\Rightarrow$  msort2 n xs = msort xs"
138 proof (induction n xs rule: msort2.induct)
139   case (1 n xs)
140   then show ?case
141     apply (auto simp: msort.simps[of xs] msort2.simps[of _ xs])
142   qed
143
144 apply auto

```

proof (prove)
goal:
No subgoals!

141.5 (9931/5309) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 416.979MB 1:41 PM

Isabelle2017 - tut07.thy

```

181 not depend on the size of the interval, e.g., iterating over the
182 interval and adding the elements separately is not allowed!
183
184
185 fun addi :: "nat  $\Rightarrow$  nat  $\Rightarrow$  intervals  $\Rightarrow$  intervals"
186 where
187   "addi i j is = undefined"
188
189 lemma addi_correct:
190   assumes "inv is" "i < j"
191   shows "inv (addi i j is)" "set of (addi i j is) = {i..j}  $\cup$  (set of is)"

```

187.1 (5137/5311) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 475.967MB 1:43 PM

Isabelle2017 - tut07.thy

```

168
169
170 fun del :: "nat  $\Rightarrow$  intervals  $\Rightarrow$  intervals"
171 where
172   "del _ _ = undefined"
173
174 lemma del_correct: "Come up with a meaningful spec yourself" oops
175
176
177
178 text < \NumHomework{Addition of Interval to Interval List}{June 1}

```

consts
del :: "nat \Rightarrow (nat \times nat) list \Rightarrow (nat \times nat) list"
Found termination order: "{}"

173.1 (4686/5311) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 420.857MB 1:43 PM

Isabelle2017 - ex06.thy

```

212 let ?xs2 = "[y←xs . -(y < x)]"
213
214 have [simp]: "sort (x#xs) = sort ?xs1 @ x # sort ?xs2"
215   using partition_correct[of "x#xs" x]
216   text <Hint: Use the lemma @{thm [source] partition_correct} and another auxiliary lemma here.>
217   by (auto simp: aux21)
218 note [simp del] = sort_key_simps
219
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221 using nat_neq_iff by blast
222 then show ?case proof cases
223 case L
224 then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber, *)
225
226 show ?thesis using L IH(1)
227   by (simp add: nth_append)
228   (* Es ist komisch, dass diese OF refl notwendig sind *)
229   thm "1.IH"
230 next

```

222.1 (6782/7833) (Isabelle, Isabelle, UTF-8-Isabelle) tmr o UG 408.923MB 1:46 PM

```

Isabelle2017 - ex06.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
219 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
220 using nat_neq_iff by blast
221 then show ?case proof cases
222 case L
223 then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber, )
224
225 show ?thesis using L IH(1)
226 by (simp add: nth_append)
227 (* Es ist komisch, dass diese OF refl notwendig sind *)
228 thm "1.IH"
229 next
230 case E
231 then have "quickselect (x # xs) k = x" by simp
232 show ?thesis by (simp add: nth_append) E
233 next
234 case G
235 then have "quickselect (x # xs) k = quickselect ?xs2 (k - length ?xs1 - 1)"
236 by simp

```

226.31 (7015/7833) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 52/912MB 1:46 PM
 debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy 13:46:51

```

Isabelle2017 - ex06.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221 using nat_neq_iff by blast
222 then show ?case proof cases
223 case L
224 then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber, )
225
226 show ?thesis using L IH(1)
227 apply (simp add: )
228 (* Es ist komisch, dass diese OF refl notwendig sind *)
229 thm "1.IH"
230 next

```

proof (prove)
 goal (1 subgoal):
 1. $[k < \text{length } [y \leftarrow xs . y < x]; \text{quickselect } [y \leftarrow xs . y < x] k = \text{sort } [y \leftarrow xs . y < x] ! k]$
 $\Rightarrow \text{sort } [y \leftarrow xs . y < x] ! k = (\text{sort } [y \leftarrow xs . y < x] @ x \# \text{sort } [y \leftarrow xs . y < x]) ! k$

227.24 (7039/7826) Input/output complete (isabelle.isabelle.UTF-8-isabelle)lmmro UG 2/902MB 1:47 PM
 debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy (modified) 13:47:44

```

Isabelle2017 - ex06.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221 using nat_neq_iff by blast
222 then show ?case proof cases
223 case L
224 then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber, )
225
226 show ?thesis using L IH(1)
227 apply (simp add: )
228 (* Es ist komisch, dass diese OF refl notwendig sind *)
229 thm "1.IH"
230 next

```

proof (prove)
 using this:
 • $k < \text{length } [y \leftarrow xs . y < x]$
 • $[k < \text{length } [y \leftarrow xs . y < x]; k < \text{length } [y \leftarrow xs . y < x]]$
 $\Rightarrow \text{quickselect } [y \leftarrow xs . y < x] k = \text{sort } [y \leftarrow xs . y < x] ! k$
 goal (1 subgoal):

226.26 (7010/7826) (isabelle.isabelle.UTF-8-isabelle)lmmro UG 2/902MB 1:47 PM
 debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy (modified) 13:47:58

```

Isabelle2017 - ex06.thy (modified)
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221 using nat_neq_iff by blast
222 then show ?case proof cases
223 case L
224 then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber, )
225
226 show ?thesis using L IH(1)
227 thm nth_append
228 apply (simp add: )
229 (* Es ist komisch, dass diese OF refl notwendig sind *)
230 thm "1.IH"

```

(?xs @ ?ys) ! ?n = (if ?n < length ?xs then ?xs ! ?n else ?ys ! (?n - length ?xs))

227.21 (7036/7847) Input/output complete (isabelle.isabelle.UTF-8-isabelle)lmmro UG 2/902MB 1:48 PM
 debian 1 2 3 4 lamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy (modified) 13:48:14

```

Isabelle2017 - ex06.thy (modified)
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
240 ohne das findet sledgehammer "manchmal" einen sehr langen smt-Beweis, aber nicht immer *)
241 proof -
242   have "length xs = length ?xs1 + length ?xs2"
243     by (simp add: sum_length_filter_compl)
244     with "1.premis" show ?thesis by simp
245   qed
246   show ?thesis using G IH(2)
247   apply (simp add: nth_append)

```

```

have k < 1 + length [y←xs . y < x] + length [y←xs . ¬ y < x]
proof (state)
this:
  k < 1 + length [y←xs . y < x] + length [y←xs . ¬ y < x]
goal (1 subgoal):
1. length [y←xs . y < x] < k ⇒ quickselect (x # xs) k = sort (x # xs) ! k

```

245,8 (7687/7832) Matches line 239: have "k < 1 + length ?xs1 + length ?xs2" (* Hier braucht man ein Hilf... (Isabelle, Isabelle, UTF-8-Isabelle) | tmr ◦ UG : : 4/875MB 1:49 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy (modified) 13:49:48

```

Isabelle2017 - ex06.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
247 by (simp add: nth_append)
248 qed
249 (*>*)
250 next
251 case 2 then show ?case by simp
252 qed
253
254

```

```

show quickselect (x # xs) k = sort (x # xs) ! k
Successful attempt to solve goal by exported rule:
(length [y←xs . y < x] < k) ⇒ quickselect (x # xs) k = sort (x # xs) ! k
proof (state)
this:
  quickselect (x # xs) k = sort (x # xs) ! k
goal:
No subgoals!

```

251,9 (7783/7833) (Isabelle, Isabelle, UTF-8-Isabelle) | tmr ◦ UG : : 63/867MB 1:50 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy 13:50:22

```

Isabelle2017 - ex06.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
210 let ?xs1 = "[y←xs . y < x]"
211 let ?xs2 = "[y←xs . ¬(y < x)]"
212
213
214 have [simp]: "sort (x#xs) = sort ?xs1 @ x # sort ?xs2"
215   using partition_correct[of "x#xs" x]
216   text <Hint: Use the lemma @[thm [source] partition_correct] and another auxiliary lemma here.>
217   by (auto simp: aux21)
218   note [simp del] = sort_key_simps
219
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221   using nat_neq_iff by blast
222   then show ?case proof cases
223     case L
224     then have "quickselect (x # xs) k = quickselect ?xs1 k" by simp (* Braucht man nicht, man sieht aber...

```

```

proof (state)
this:
  · sort_key ?f [] = []
  · sort_key ?f (?x # ?xs) = insert_key ?f ?x (sort_key ?f ?xs)

```

219,1 (6675/7833) (Isabelle, Isabelle, UTF-8-Isabelle) | tmr ◦ UG : : 86/859MB 1:51 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy 13:51:00

```

Isabelle2017 - ex06.thy
File Edit Search Markers Folding View Utilities Magros Plugins Help
ex06.thy (~/lehre/FDS/Private/SS18/Exercises/ex06/)
199 proof (induction xs k rule: quickselect.induct)
200   case (1 x xs k)
201
202   text <Note: To make the induction hypothesis more readable,
203     you can collapse the first two premises of the form <?x=...>
204     by reflexivity:>
205     note IH = "1.IH"[OF refl refl]
206
207   text <Insert your proof here!>
208
209   (*<*)
210
211   let ?xs1 = "[y←xs . y < x]"
212   let ?xs2 = "[y←xs . ¬(y < x)]"
213

```

201,1 (6131/7833) (Isabelle, Isabelle, UTF-8-Isabelle) | tmr ◦ UG : : 5/859MB 1:51 PM
 debian 1 2 3 4 iamlich@lapnikow10: ~/lehre/FDS/SS1... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy 13:51:22

```
Isabelle2017 - ex06.thy
File Edit Search Markers Folding View Utilities Macros Plugins Help
ex06.thy (~/.lehre/FDS/Private/SS18/Exercises/ex06/)
208
209
210
211 let ?xs1 = "[y←xs . y < x]"
212 let ?xs2 = "[y←xs . ¬(y < x)]"
213
214 have [simp]: "sort (x#xs) = sort ?xs1 @ x # sort ?xs2"
215   using partition_correct[of "x#xs" x]
216   text <Hint: Use the lemma @{thm [source] partition_correct} and another auxiliary lemma here.>
217   by (auto simp: aux21)
218 note [simp del] = sort_key_simps
219
220 consider (L) "k < length ?xs1" | (E) "k = length ?xs1" | (G) "k > length ?xs1"
221   using nat_neq_iff by blast
222 then show ?case proof cases
223
proof (prove)
using this:
  sort (x # xs) = sort [xa←x # xs . xa < x] @ sort [xa←x # xs . ¬ xa < x]
end
end

Output Query Sledgehammer Symbols
218.1 (6640/7833) (isabelle.isabelle.UTF-8+isabelle)1m r o UG 396/350MB 1:51 PM
debian 1 2 3 4 iamnich@lapnikow10: ~/.lehre/FDS/SS18/... Isabelle2017 - tut07.thy Isabelle2017 - ex06.thy 13:51:55
```