

Title: Simon: Programmiersprachen (05.05.2014)

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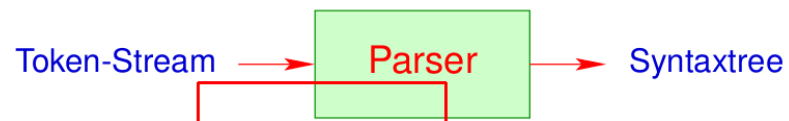
Pages: 42

Topic:

Syntactic Analysis

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Syntactic Analysis



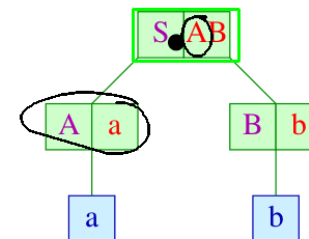
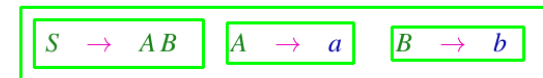
- Syntactic analysis tries to integrate Tokens into larger program units.



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Item Pushdown Automaton – Example

Our example:

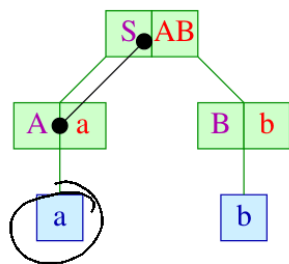


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Item Pushdown Automaton – Example

Our example:

$S \rightarrow AB$ $A \rightarrow a$ $B \rightarrow b$

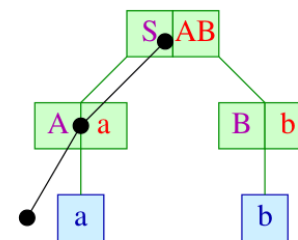


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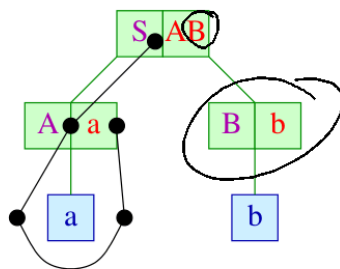


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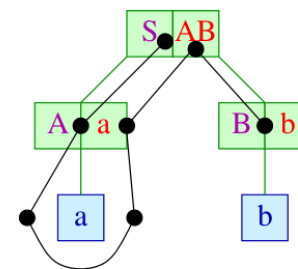


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Item Pushdown Automaton – Example

Our example:

$S \rightarrow AB$ $A \rightarrow a$ $B \rightarrow b$



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Item Pushdown Automaton – Example

We add another rule $S' \rightarrow S$ for initialising the construction:

Start state:

$[S' \rightarrow \bullet S]$

End state:

$[S' \rightarrow S \bullet]$

Transition relations:

$[S' \rightarrow \bullet S]$	ϵ	$[S' \rightarrow \bullet S] [S \rightarrow \bullet AB]$
$[S \rightarrow \bullet AB]$	ϵ	$[S \rightarrow \bullet AB] [A \rightarrow \bullet a]$
$[A \rightarrow \bullet a]$	a	$[A \rightarrow a \bullet]$
$[S \rightarrow \bullet AB] [A \rightarrow a \bullet]$	ϵ	$[S \rightarrow A \bullet B]$
$[S \rightarrow A \bullet B]$	ϵ	$[S \rightarrow A \bullet B] [B \rightarrow \bullet b]$
$[B \rightarrow \bullet b]$	b	$[B \rightarrow b \bullet]$
$[S \rightarrow A \bullet B] [B \rightarrow b \bullet]$	ϵ	$[S \rightarrow AB \bullet]$
$[S' \rightarrow \bullet S] [S \rightarrow AB \bullet]$	ϵ	$[S' \rightarrow S \bullet]$

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Item Pushdown Automaton

Discussion:

- The **expansions** of a computation form a **leftmost derivation**
- Unfortunately, the expansions are chosen **nondeterministically**
- For proving correctness of the construction, we show that for every item $[A \rightarrow \alpha \bullet B \beta]$ the following holds:

$$([A \rightarrow \alpha \bullet B \beta], w) \vdash^* ([A \rightarrow \alpha B \bullet \beta], \epsilon) \quad \text{iff} \quad B \rightarrow^* w$$

- **LL-Parsing** is based on the item pushdown automaton and tries to make the expansions deterministic ...

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Item Pushdown Automaton

The item pushdown automaton M_G^L has three kinds of transitions:

Expansions: $([A \rightarrow \alpha \bullet B \beta], \epsilon, [A \rightarrow \alpha \bullet B \beta] [B \rightarrow \bullet \gamma])$ for $A \rightarrow \alpha B \beta, B \rightarrow \gamma \in P$

Shifts: $([A \rightarrow \alpha \bullet a \beta], a, [A \rightarrow \alpha a \bullet \beta])$ for $A \rightarrow \alpha a \beta \in P$

Reduces: $([A \rightarrow \alpha \bullet B \beta] [B \rightarrow \gamma \bullet], \epsilon, [A \rightarrow \alpha B \bullet \beta])$ for $A \rightarrow \alpha B \beta, B \rightarrow \gamma \in P$

Items of the form: $[A \rightarrow \alpha \bullet]$ are also called **complete**

The item pushdown automaton shifts the dot once around the derivation tree ...

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The item pushdown automaton shifts the dot once around the derivation tree ...

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Item Pushdown Automaton

Beispiel: $S \rightarrow \epsilon \mid aSb$

The transitions of the according Item Pushdown Automaton:

0	$[S' \rightarrow \bullet S]$	ϵ	$[S' \rightarrow \bullet S]$	$[S \rightarrow \bullet]$
1	$[S' \rightarrow \bullet S]$	ϵ	$[S' \rightarrow \bullet S]$	$[S \rightarrow \bullet aSb]$
2	$[S \rightarrow \bullet aSb]$	a	$[S \rightarrow a \bullet Sb]$	
3	$[S \rightarrow a \bullet Sb]$	ϵ	$[S \rightarrow a \bullet Sb]$	$[S \rightarrow \bullet]$
4	$[S \rightarrow a \bullet Sb]$	ϵ	$[S \rightarrow a \bullet Sb]$	$[S \rightarrow \bullet aSb]$
5	$[S \rightarrow a \bullet Sb]$	$[S \rightarrow \bullet]$	ϵ	$[S \rightarrow a S \bullet b]$
6	$[S \rightarrow a \bullet Sb]$	$[S \rightarrow aSb \bullet]$	ϵ	$[S \rightarrow a S \bullet b]$
7	$[S \rightarrow a S \bullet b]$	b	$[S \rightarrow aSb \bullet]$	
8	$[S' \rightarrow \bullet S]$	$[S \rightarrow \bullet]$	ϵ	$[S' \rightarrow S \bullet]$
9	$[S' \rightarrow \bullet S]$	$[S \rightarrow aSb \bullet]$	ϵ	$[S' \rightarrow S \bullet]$

Conflicts arise between the transitions (0, 1) and (3, 4), resp..

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Topdown Parsing

Problem: Conflicts between the transitions prohibit an implementation of the item pushdown automaton as deterministic pushdown automaton.

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Topdown Parsing

Problem:

Conflicts between the transitions prohibit an implementation of the item pushdown automaton as deterministic pushdown automaton.

Idee 1: GLL Parsing

For each conflict, we create a virtual copy of the complete stack and continue computing in parallel.

Idee 2: Recursive Descent & Backtracking

Depth-first search for an appropriate solution.

Idee 3: Recursive Descent & Lookahead

Conflicts are resolved by considering a lookup of the next input symbol.

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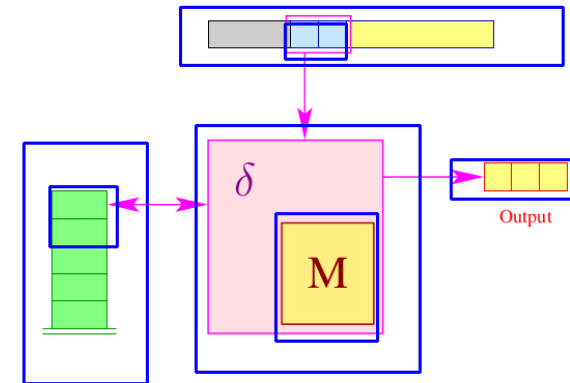
Topdown Parsing

Idee:

- Emanate from the item pushdown automaton
- Consider **the next** symbol to determine the appropriate rule for the next expansion
- A grammar is called **LL(1)** if a unique choice is always possible

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Structure of the LL(1)-Parser:



- The parser accesses a frame of length 1 of the input;
- it corresponds to an item pushdown automaton, essentially;
- table $M[q, w]$ contains the rule of choice.

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Topdown Parsing

Idee:

- Emanate from the item pushdown automaton
- Consider **the next** symbol to determine the appropriate rule for the next expansion
- A grammar is called **LL(1)** if a unique choice is always possible

Definition:

A reduced grammar is called **LL(1)** if for each two distinct rules $A \rightarrow \alpha$, $A \rightarrow \alpha' \in P$ and each derivation $S \rightarrow_L^* u A \beta$ with $u \in T^*$ the following is valid.

$$\text{First}_1(\alpha \beta) \cap \text{First}_1(\alpha' \beta) = \emptyset$$



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Topdown Parsing

Example 1:

$S \rightarrow \text{if } (E) S \text{ else } S \mid \text{while } (E) S \mid E;$
 $E \rightarrow \text{id}$

is $LL(1)$, since $\text{First}_1(E) = \{\text{id}\}$

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Topdown Parsing

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Example 2:

$S \rightarrow \text{if } (E) S \text{ else } S \mid \text{if } (E) S \mid \text{while } (E) S \mid E;$
 $E \rightarrow \text{id}$

... is not $LL(k)$ for any $k > 0$.

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Lookahead Sets

Definition:

For a set $L \subseteq T^*$ we define:

$$\text{First}_1(L) = \{\epsilon \mid \epsilon \in L\} \cup \{u \in T \mid \exists v \in T^* : uv \in L\}$$

Example: $S \rightarrow \epsilon \mid S \rightarrow aSb$ $\text{First}_1(S)$

ϵ	\rightarrow	ϵ
ab	\rightarrow	a
$aabb$	\rightarrow	a
$aaabbb$	\rightarrow	a
\vdots		

$\text{First}_1(S) = \{\epsilon, a\}$

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Lookahead Sets

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Example:

ϵ
ab
$aabb$
$aaabbb$

the prefixes of length 1

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Lookahead Sets

Arithmetics:

$\text{First}_1(_)$ is compatible with union and concatenation:

$$\begin{aligned} \text{First}_1(\emptyset) &= \emptyset \\ \text{First}_1(L_1 \cup L_2) &= \text{First}_1(L_1) \cup \text{First}_1(L_2) \\ \text{First}_1(L_1 \cdot L_2) &= \text{First}_1(\text{First}_1(L_1) \cdot \text{First}_1(L_2)) \\ &:= \text{First}_1(L_1) \odot \text{First}_1(L_2) \end{aligned}$$

1 – concatenation

Observation:

Let $L_1, L_2 \subseteq T \cup \{\epsilon\}$ with $L_1 \neq \emptyset \neq L_2$. Then:

$$L_1 \odot L_2 = \begin{cases} L_1 & \text{if } \epsilon \notin L_1 \\ (L_1 \setminus \{\epsilon\}) \cup L_2 & \text{otherwise} \end{cases}$$

If all rules of G are productive, then all sets $\text{First}_1(A)$ are non-empty.

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Lookahead Sets

For $\alpha \in (N \cup T)^*$ we are interested in the set:

$$\text{First}_1(\alpha) = \text{First}_1(\{w \in T^* \mid \alpha \rightarrow^* w\})$$

Idea: Treat ϵ separately: F_ϵ

- Let $\text{empty}(X) = \text{true}$ iff $X \rightarrow^* \epsilon$.
- $F_\epsilon(X_1 \dots X_m) = \bigcup_{i=1}^m F_\epsilon(X_i)$ if $\text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_{j-1})$

We characterize the ϵ -free First_1 -sets with an inequality system:

$$\begin{aligned} F_\epsilon(a) &= \{a\} & \text{if } a \in T \\ F_\epsilon(A) &\supseteq F_\epsilon(X_j) & \text{if } A \rightarrow X_1 \dots X_m \in P, \\ & & \text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_{j-1}) \end{aligned}$$

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Lookahead Sets

for example...

$$\begin{array}{l} E \rightarrow E + T \quad | \quad T \\ T \rightarrow T * F \quad | \quad F \\ F \rightarrow (E) \quad | \quad \text{name} \quad | \quad \text{int} \end{array}$$

with $\text{empty}(E) = \text{empty}(T) = \text{empty}(F) = \text{false}$

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Lookahead Sets

for example...

$$\begin{array}{l|l} E \rightarrow E+T & T \\ T \rightarrow T*F & F \\ F \rightarrow (E) & \text{name} \quad | \quad \text{int} \end{array}$$

with $\text{empty}(E) = \text{empty}(T) = \text{empty}(F) = \text{false}$

... we obtain:

$$\begin{array}{l} F_\epsilon(S') \supseteq F_\epsilon(E) \quad F_\epsilon(E) \supseteq F_\epsilon(E) \\ F_\epsilon(E) \supseteq F_\epsilon(T) \quad F_\epsilon(T) \supseteq F_\epsilon(T) \\ F_\epsilon(T) \supseteq F_\epsilon(F) \quad F_\epsilon(F) \supseteq \{ (, \text{name}, \text{int}) \} \end{array}$$

Fast Computation of Lookahead Sets

Observation:

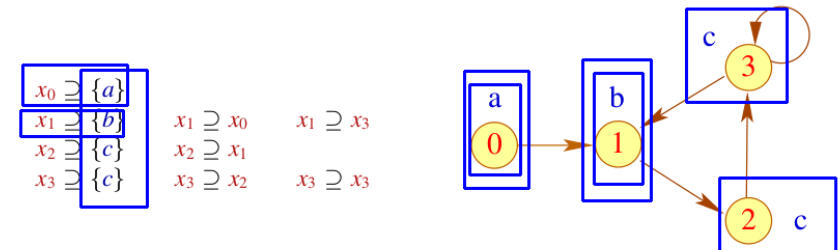
- The form of each inequality of these systems is:

$$x \supseteq y \quad \text{resp.} \quad x \supseteq d$$

for variables x, y und $d \in D$.

- Such systems are called **pure unification problems**
- Such problems can be solved in **linear** space/time.

for example: $D = 2^{\{a,b,c\}}$



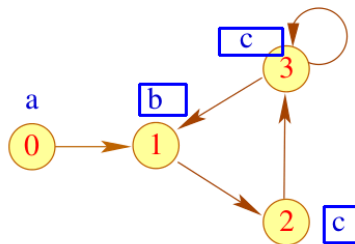
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Fast Computation of Lookahead Sets



Frank DeRemer
& Tom Pennello



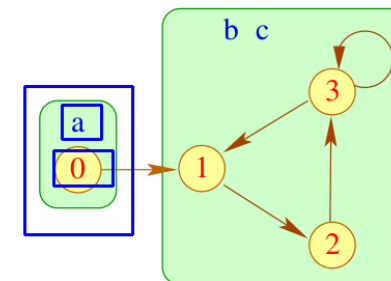
Proceeding:

- Create the **Variable dependency graph** for the inequality system.

Fast Computation of Lookahead Sets



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& Tom Pennello



Proceeding:

- Create the **Variable dependency graph** for the inequality system.
- Whithin a **strongly connected component** (\rightarrow Tarjan) all variables have the same value
- Is there no ingoing edge for an SCC, its value is computed via the smallest upper bound of all values within the SCC

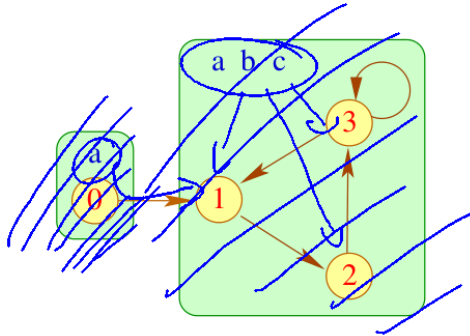
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Fast Computation of Lookahead Sets



Frank DeRemer & Tom Pennello



Proceeding:

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- Whithin a **strongly connected component** (\rightarrow Tarjan) all variables have the same value
- Is there no ingoing edge for an SCC, its value is computed via the smallest upper bound of all values within the SCC
- In case of ingoing edges, their values are also to be considered for the upper bound

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Item Pushdown Automaton as LL(1)-Parser

back to the example:

$S \rightarrow \epsilon \mid aSb$

The transitions in the according Item Pushdown Automaton:

0	$[S' \rightarrow \bullet S]$	ϵ	$[S' \rightarrow \bullet S]$ $[S \rightarrow \bullet]$
1	$[S' \rightarrow \bullet S]$	ϵ	$[S' \rightarrow \bullet S]$ $[S \rightarrow \bullet aSb]$
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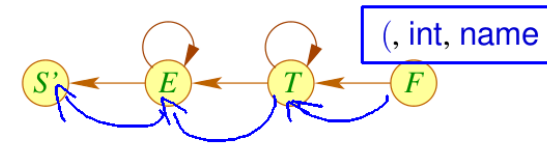
Conflicts arise between transations (0,1) or (3,4) resp..

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Fast Computation of Lookahead Sets

... for our example grammar:

First₁ :



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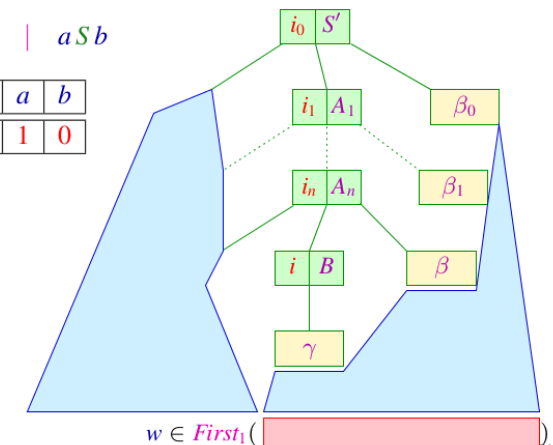
Item Pushdown Automaton as LL(1)-Parser

Is G an $LL(1)$ -grammar, we can index a lookahead-table with items and nonterminals:

We set $M[B, w] = i$ exactly if (B, i) is the rule $B \rightarrow \gamma$ and: $w \in \text{First}_1(\gamma) \odot \cup \{ \text{First}_1(\beta) \mid S' \rightarrow^* uB\beta \}$.

... for example: $S \rightarrow \epsilon \mid aSb$

	ϵ	a	b
S	0	1	0



$w \in \text{First}_1(\text{...})$

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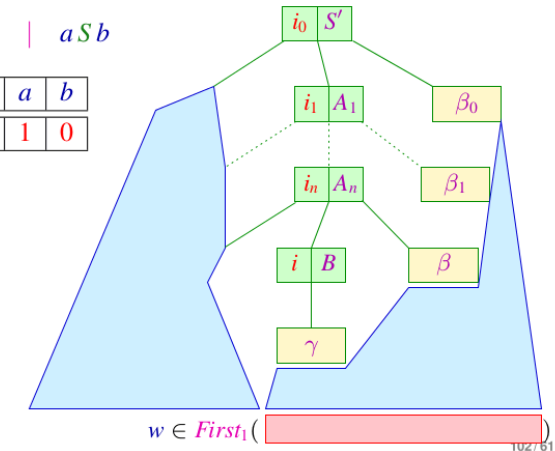
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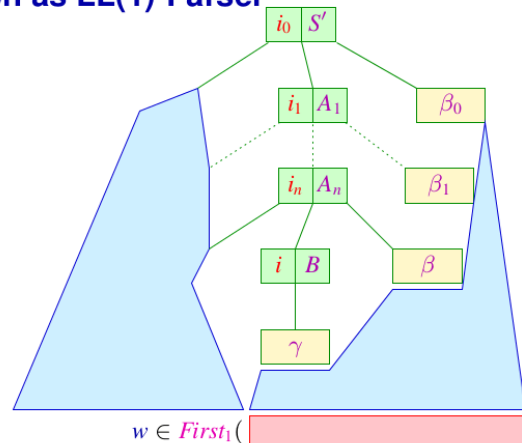
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	ϵ	a	b
S	0	1	0



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Item Pushdown Automaton as LL(1)-Parser



Inequality system for $\text{Follow}_1(B) = \bigcup \{ \text{First}_1(\beta) \mid S' \rightarrow \overset{*}{\rightarrow} uB\beta \}$

$\text{Follow}_1(S) \supseteq \{ \epsilon \}$	
$\text{Follow}_1(B) \supseteq F_\epsilon(X_j)$	if $A \rightarrow \alpha B X_1 \dots X_m \in P,$ $\text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_{j-1})$
$\text{Follow}_1(B) \supseteq \text{Follow}_1(A)$	if $A \rightarrow \alpha B X_1 \dots X_m \in P,$ $\text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_m)$

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Item Pushdown Automaton as LL(1)-Parser

For example: $S \rightarrow \epsilon \mid aSb$

The transitions of the according Item Pushdown Automaton:

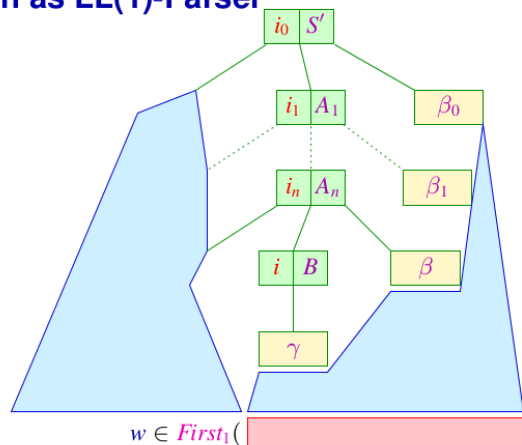
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Lookahead table:

	ϵ	a	b
S	0	1	0

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Inequality system for $\text{Follow}_1(B) = \bigcup \{ \text{First}_1(\beta) \mid S' \xrightarrow{*} u B \beta \}$

$$\begin{aligned} \text{Follow}_1(S) &\supseteq \{ \epsilon \} \\ \text{Follow}_1(B) &\supseteq F_c(X_j) && \text{if } A \rightarrow \alpha B X_1 \dots X_m \in P, \\ & && \text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_{j-1}) \\ \text{Follow}_1(B) &\supseteq \text{Follow}_1(A) && \text{if } A \rightarrow \alpha B X_1 \dots X_m \in P, \\ & && \text{empty}(X_1) \wedge \dots \wedge \text{empty}(X_m) \end{aligned}$$

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Topdown-Parsing

Discussion

- A practical implementation of an $LL(1)$ -parser via **recursive Descent** is a straight-forward idea
- However, **only a subset** of the deterministic contextfree languages can be read this way.
- **Solution:** Going from $LL(1)$ to $LL(k)$
- The size of the occurring sets is rapidly increasing with larger k
- Unfortunately, even $LL(k)$ parsers are not sufficient to accept all deterministic contextfree languages.
- In practical systems, this often motivates the implementation of $k = 1$ only ...

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End of presentation. Click to exit.